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FINDING THE IMPORTANT FACTORS IN BATTLE OUTCOMES: A STATISTICAL EXPLORATION OF DATA FROM MAJOR BATTLES

by

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This study explores important factors in battle outcomes through a statistical analysis of data from major historical battles. The data set of CDB90FT has been made available and documented by the Center for Army Analysis (CAA). The quality of the historical data is good. There are 660 battles listed in the data set containing over 140 numerical features for each battle. The earliest battle in the data set is the Netherlands' War of Independence in 1600, while the last one is from the Israel-Lebanon War in 1982. The data set contains many interesting facts on the battles including the initial strengths, the total strengths, the number of casualties, the lengths of the front lines, terrain features, command capability of leaders, weather conditions, etc. The approach is to use the data set as the basis for an objective and scientific comprehensive analysis, seeking patterns, trends, and relationships in combat. After making campaign-wise grouping and analysis, it is found that the Force Ratio is a valid estimator of the battle outcome. In addition, the Casualty Rate has declined steadily over the past four centuries while Dispersion has increased.

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FINDING THE IMPORTANT FACTORS IN BATTLE OUTCOMES: A STATISTICAL EXPLORATION OF DATA FROM MAJOR BATTLES

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ABSTRACT

This study explores important factors in battle outcomes through a statistical analysis of data from major historical battles. The data set of CDB90FT has been made available and documented by the Center for Army Analysis (CAA). The quality of the historical data is good. There are 660 battles listed in the data set containing over 140 numerical features for each battle. The earliest battle in the data set is the Netherlands' War of Independence in 1600, while the last one is from the Israel-Lebanon War in 1982. The data set contains many interesting facts on the battles including the initial strengths, the total strengths, the number of casualties, the lengths of the front lines, terrain features, command capability of leaders, weather conditions, etc. The approach is to use the data set as the basis for an objective and scientific comprehensive analysis, seeking patterns, trends, and relationships in combat. After making campaign-wise grouping and analysis, it is found that the Force Ratio is a valid estimator of the battle outcome. In addition, the Casualty Rate has declined steadily over the past four centuries while Dispersion has increased.

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EXECUTIVE SUMMARY

From historical records of civilization, analysts have struggled to find some order and predictability in the confusion of war. Using their practical experience with past conflicts and using whatever other information was available, they have sought general rules about the nature of war that could help them prevail in future conflicts. Although no one can possibly know what the next war will be like, there can be no doubt, however, that some of the emotional, conceptual, and intellectual aspects of combat over the ages are consistent over time. These aspects may give clues about what we might face in the future.

Students of military art and military science have searched for fundamental laws or theories that would explain the interactions of military forces in combat and the outcomes of battles. The annals of military experience served as a guide for those students to get answers to the questions about the basic laws of combat. The phenomenon of war has continued to fascinate scholars, particularly those trained in military skills. Those scholars tried to gain insights into the concept of warfare and to identify the patterns of war. Even when the results of a battle appear to have deviated from an identified pattern, an analyst generally reveals that this deviation is due to the operation of another pattern upon the circumstances of the battle.

Not only does an analysis of ancient battles involve the telling of a story, it also evaluates what the story means. So, the analyst must develop combat hypotheses by means of patterns discerned from studying large quantities of combat data. This thesis uses the CDB90FT data as the basis for an initial objective, scientific, and comprehensive analysis, seeking patterns, trends, and relationships, as well as comparing the results to

previous studies. Alternative hypotheses are tested against the data. As more data is available for study, confidence in the validity of the hypotheses increases.

First, the concept of Force Ratio (FR) is analyzed for the campaign-wise groupings of the CDB90FT data set. The analysis seeks general trends concerning FR in the campaigns and formulates the relations between trends and battle outcomes. The data set is divided into 17 campaigns that comprise 552 battles out of the 660 battles in the actual data set.

As a gross measure for campaign planning, FR is useful and stands up quite well under historical scrutiny. As a basis for forecasting battle outcomes, however, it seems to be more probabilistic than deterministic. As such, the FR is less reliable in terms of predicting the battle outcome. The simple formula for FR is:

$$FR = A / D \tag{E.1}$$

where A is the total force strength of the attacker in manpower,

D is the total force strength of the defender in manpower.

Second, the concept of "dispersion" is analyzed for the campaign-wise grouping of the CDB90FT data set. The analysis seeks general trends concerning campaigns. Since the specific data of "depth" in the combat is available for only six campaigns in the data set of CDB90FT, the analysis covers only those campaigns, which are the Napoleonic Wars, the American Civil War, the Franco-Russian War, World War I (WWI), World War II (WWII), and the Arab-Israel War of 1973. Understanding the concept that dispersion includes more than the physical area that is occupied clarifies the concept of density to an analyst. A mental picture of dispersion is provided by looking at the average

troop density on the battlefield in terms of men per square kilometer. The simple formulas for the density and the dispersion are:

$$\text{Density} = (\text{Total strength of the army}) / (\text{Total area occupied}) \tag{E.2}$$

$$\text{Dispersion} = 1 / (\text{Density}) \tag{E.3}$$

The third subject explained in this thesis is the concept of casualty rate, specifically the daily casualty rate (DCR). Among many interactions relating to casualty, three factors are analyzed:

1. Historical trends in the DCR,
2. The size of the unit versus the DCR,
3. The battles of each campaign in time sequence versus the DCR.

The figures and the tables in this Chapter are based on the CDB90FT data set, which is the compilation of 660 major battles. There will be three different analyses, each corresponding to a single factor of interest listed above. The average DCR's of the attacker and the defender are used to determine the historical trends in the casualty rate. After we calculate the DCR for each individual battle, the results are averaged, a number representing the average DCR for both the attacker and the defender in the campaign is calculated.

This study explores important factors in battle outcomes through a statistical analysis of data from major historical battles. The data set of CDB90FT [Ref 1.7] has been made available and documented by the Center for Army Analysis (CAA). The quality of the historical data is good. There are 660 battles listed in the data set containing over 140 numerical features for each battle. The earliest battle in the data set is the Netherlands' War of Independence in 1600, while the last one is from the Israel-Lebanon

War in 1982. The data set contains many interesting facts on the battles including the initial strengths, the total strengths, the number of casualties, the lengths of the front lines, terrain features, command capability of leaders, weather conditions, etc.

The findings from this research include:

- Greater dispersion of combat troops on the battlefield is a reason for a decrease in casualties despite an increase in weapons lethality. This greater dispersion has occurred in response to increasing lethality of new weapons. As lethality increased, tactics, such as increasing the dispersion of combat forces, were adopted to minimize the effectiveness of the enemy's weapons [Figure E.1].

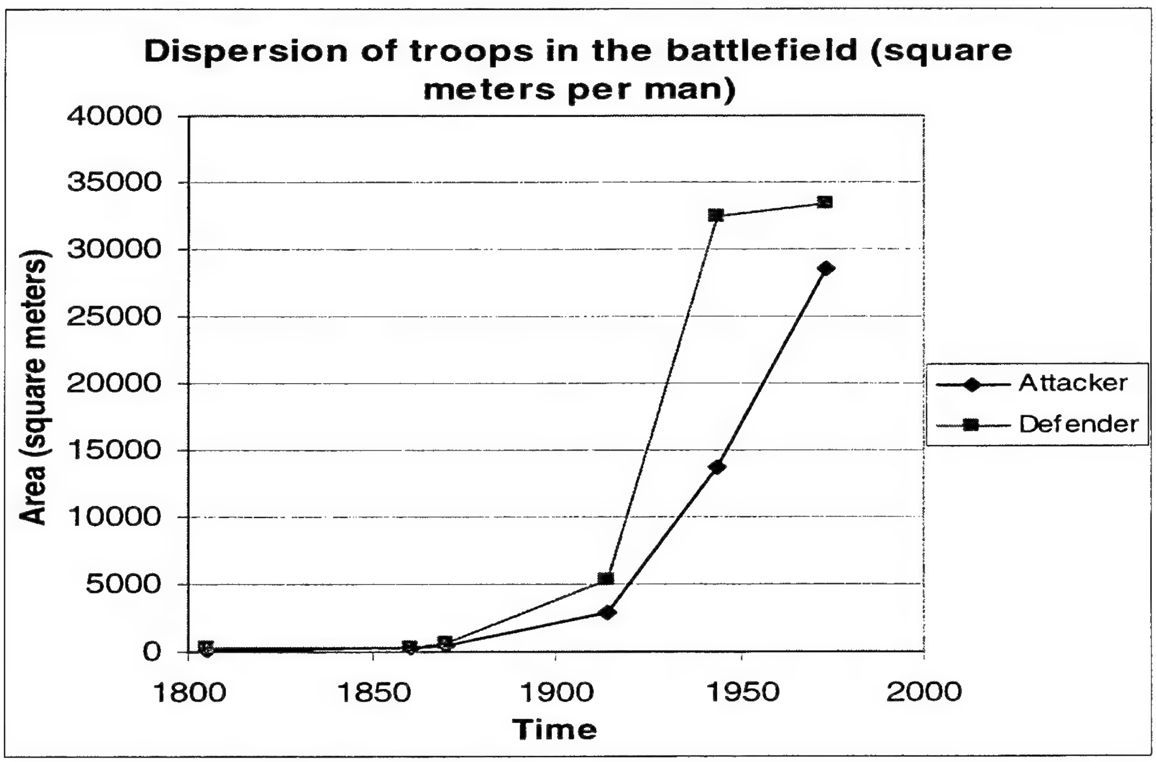

Figure E.1. The Dispersions of the Campaigns. Note that the unit area is m^2.

• Three very general patterns are evident in the historical casualty data that has been analyzed. Casualty rates have declined generally over the past four centuries and almost leveled off at the rates experienced in WWII and Arab–Israel Wars [Figure E.2].

• Casualty rates of the attackers are almost always lower than those of the defenders. Also, the CR values decrease as the unit size in the battle increases.

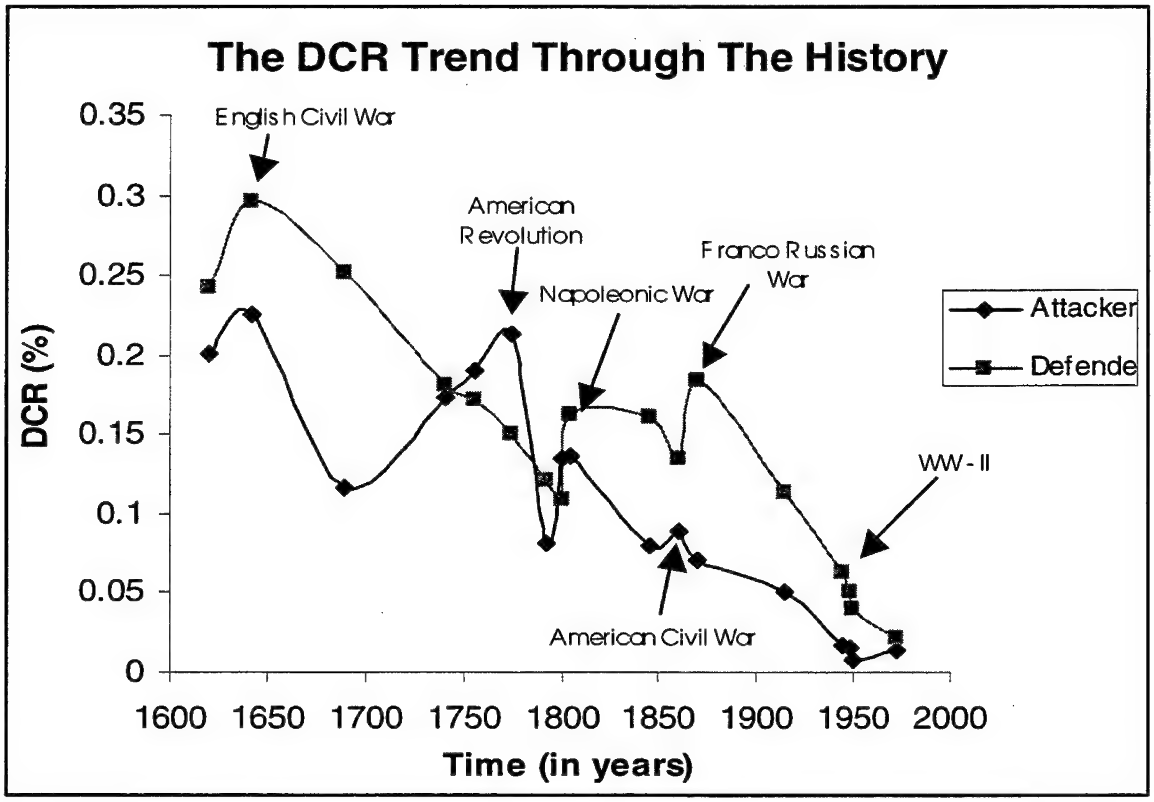

Figure E.2. The Average DCR's of the Attackers and the Defenders in the Campaigns.

• Even though it is more probabilistic than other battle outcome predictors [Ref 3.8], the Force Ratio is a valid estimator of the battle outcome.

• Despite some slight differences among probability of winning values corresponding to specific FR values of the data set, the general trend remains applicable for the overall analysis of the campaigns, emphasizing that the P(Attacker wins given FR) value increases as the FR value increases.

LIST OF SYMBOLS, ACRONYMS AND ABBREVIATIONS

- FR : Force Ratio
- CR : Casualty Rate
- DCR : Daily Casualty Rate
- A : Attacker
- D : Defender
- WWI : World War I
- WWII : World War II

I. INTRODUCTION

From historical records of civilization, analysts have struggled to find some order and predictability in the confusion of war. Using their practical experience with past conflicts and using whatever other information was available, they have sought general rules about the nature of war that could help them prevail in future conflicts.

Although no one can possibly know what the next war will be like, there can be no doubt, however, that some of the emotional, conceptual, and intellectual aspects of combat over the ages are consistent over time. These aspects may give clues about what we might face in the future.

A. GENERAL INTRODUCTION

Not only does an analysis of ancient battles involve the telling of a story, it also evaluates what the story means. So, the analyst must develop combat hypotheses by means of patterns discerned from studying large quantities of combat data. The approach in this thesis is to use the CDB90FT data as the basis for an objective, scientific, and comprehensive analysis, seeking patterns, trends, and relationships, as well as comparing the results to previous studies. Alternative hypotheses will be tested against the data. As more data is available for study, confidence in the validity of the hypotheses increases.

Some previous studies of combat using historical data include: Bracken [Ref 1.1], Fricker [Ref 1.2], Clemens [Ref 1.3], Hartley and Helmbold [Ref 1.4], and Turker [Ref 1.5]. We will contrast what we find in the CDB90FT data set with the findings of these authors.

For the Ardennes campaign, Bracken formulates four different models [Ref 1.1], which are variations of basic Lanchester equations and estimates their parameters by a

grid search using the first ten days of the Ardennes Campaign of World War II (December 15, 1944 through January 16, 1945). Bracken's models are homogeneous. Tanks, armored personnel carriers, artillery, and manpower are aggregated with weights representing the relative effectiveness of the weapon systems. This type of aggregation yields a single measure of strength for each of the Allied and German forces. This method is used to measure combat power and to calculate losses. His models treat combat forces and the total forces (i.e., both support forces and the combat forces) in the campaign separately.

Bracken's main conclusions are:

- The Lanchester linear model best fits the Ardennes campaign data in all four cases.
- When combat forces are considered, Allied individual effectiveness is greater than German individual effectiveness. When total forces are considered, individual effectiveness is the same for both sides.
- There is an attacker advantage.

Fricker [Ref 1.2] revisited Bracken's modeling of the Ardennes campaign of World War II [Ref 1.1] and also used the Lanchester equations. Fricker found that none of the basic Lanchester models fit the data. His study is different from Bracken's study in several ways. Fricker's study:

- Uses linear regression to fit the model parameters.
- Uses the total body of data from the entire campaign, while Bracken used only the first 10 days of the data from the Ardennes Campaign.
- Includes air sortie data.

Clemens' analysis [Ref 1.3] examines the validity of the Lanchester Models as they are applied to modern warfare. The models in his study are also based upon basic Lanchester Equations. The analysis is an extension of Bracken's [Ref 1.1] and Fricker's [Ref 1.2] analyses of the Ardennes Campaign and applies the Lanchester models to the Battle of Kursk data.

Clemens uses two estimation techniques, linear regression and Newton-Raphson iteration. The analysis also explores the presented model in matrix form and compares the matrix solution to the scalar solution. In his study he concludes that [Ref 1.3]:

- Neither the Lanchester linear nor the Lanchester square model fits the data.
- The Lanchester logarithmic model in both scalar and matrix form fits better than the Lanchester linear and square models.
- Basic Lanchester Equations do not give the best fit for the data.

Hartley and Helmbold's study [Ref 1.4] focuses on validating the homogenous Lanchester square law by using historical combat data. Since validating a model means testing it in a real life context, Hartley and Helmbold test Lanchester's square law using data from the Inchon-Seoul campaign of the Korean War.

Hartley and Helmbold use three analysis techniques to examine the data; linear regression, the Akaike Information Criterion (AIC), and Bozdogan's consistent AIC (CAIC). The results of the study are [Ref 1.4]:

- The data do not fit a constant coefficient Lanchester square law.
- The data better fit a set of three separate battles (one distinct battle every six or seven days). However, the data fit a set of three constant casualty-model battles just as well.

- Lanchester square law is not a proven attrition algorithm for warfare, but neither can it be completely discounted.
- More real combat data are needed to validate any proposed attrition law, such as the Lanchester square law.

Turker's thesis [Ref 1.5] extends the previous research by validating Lanchester's equations with real data. Turker's thesis examines how the various derivatives of Lanchester's equations fit the newly compiled database on the Battle of Kursk. The results are contrasted with earlier studies on the Ardennes campaign. It turns out that a wide variety of models fit the data about as well. Unfortunately, none of the basic Lanchester equations fit very well.

As the analysis of historical data has accumulated and evolved, some numerical outcomes and fundamentals of warfare have either settled as fundamental rules or have been opened to discussions and led to the contributions of later analysts. These concepts include [Ref 1.6]:

- the three-to-one ratio,
- advance rates in combat,
- attrition in combat,
- fractional exchange ratio,
- casualty rates,
- indicators of a result of a battle.

This research extends the numerical fundamentals of warfare, some of which were listed above, by fitting functional forms to a newly released data set of 660 major battles. The main areas of interest are:

- the three-to-one ratio and its validity,
- indicators of battle outcomes,
- what causes casualties,
- validity of fractional exchange ratios,
- attrition rates,
- relationships among various numerical features of warfare such as terrain, strength, casualties, fire power, geographical diameters, leadership, weather conditions, etc.

The data set that we use is the CDB90FT [Ref 1.7], which has been made available and documented by the Center for Army Analysis (CAA). The quality of the historical data is good. There are 660 battles listed in the data set containing over 140 numerical features for each entry. The earliest battle in the data set is the Netherlands' War of Independence in 1600, while the last one is from the Israel-Lebanon War in 1982 [Ref 1.7]. The information in the data set includes the initial strengths, the total strengths, the number of casualties, the lengths of front lines, terrain features, command capability of leaders, weather conditions and many more features.

Some of the data about battles in the CDB90FT is either missing or unavailable. Thus, it is important to reorganize the data set in a way that will not lead us to misinterpretations. One way of reorganizing the data set, maybe the easiest way, is to extract columns and rows that are dominated by missing data cells. However, elimination of rows and columns might cause a loss of information. Despite domination by void cells, some available data will be lost if all columns and rows with missing data are extracted from the database.

Having understood the danger of losing information, we reorganize the data set by estimating some numbers in the void cells that will represent the missing data, while allowing one to study the other available data in the same row or column. At this point, it is essential to note that these numbers contribute nothing to the analysis; they only show that particular data is missing or unavailable. As a result, this research is based on campaign-wise grouping and modeling of data.

B. RESEARCH QUESTIONS

Essential research questions to be answered are:

1) What relates to winning?
2) What relates to casualties?
3) Is the three-to-one ratio to attack sensible?
4) Do attackers suffer more casualties?
5) What is the validity of the fractional exchange ratio?
6) How successful are the attackers?
7) What patterns exist among various features of the battlefield?
8) Are the findings consistent across time and the type of battle?

There are quite satisfactory answers for seven of the questions listed above, except for the first question, which is the broadest of all questions. However, the very first question is partially answered. Every research study of combat could formulate thousands of variables of the battlefield, if not millions, to determine what relates to winning in combat. Since each variable of combat has its own effect on the outcome of the conflict, it is very difficult for the analyst to evaluate how the specific variables of

combat, for example, dispersion, define the result of the battle. Thus, this study partially answers the first question, within its limits.

C. SCOPE OF THE THESIS

Appropriate forms of general curve fits and modals available in software, such as S-Plus [Ref 1.8], are used to analyze the CDB90FT data. Furthermore, the validity and effectiveness of results are questioned. Thus, the scope of this analysis consists of:

1) A collective usage of curve fittings/plotting and regression methods.
2) Interpretation and analysis of patterns or probable results out of the data.
3) Referencing to past analyses' results.
4) Conclusions about the results of the analyses.

It is hoped that this initial study on the database will help facilitate analysis for future researchers. Thus, some effort is made to partition and describe various features of the CDB90FT data set.

D. ORGANIZATION OF THE THESIS

The thesis is in a hierarchical order as a whole. The subjects and results of each chapter are consistent with those of other chapters in the thesis so that there is not a contradiction among the stated results. Actually, some results of previous chapters in the thesis are referenced in the later chapters that use those findings as guidelines to their results.

First, we examine the basic concepts of the research from a literature review so that we can understand the subject of the thesis clearly. By searching, exploring, and reviewing the CDB90FT data set, we identify quantities that are characteristic of the phenomenon to be measured. Since all entries of the data set are not in numerical form,

we convert them into quantitative forms in a way that is rational for analysis. In this phase, we decide on which features of the data set deserve more exploration.

Data analysis or derivation of empirical relationships among the measurable features is the main phase of our study. Prior to this main phase, the data set is ready for analysis. The data is analyzed in software, such as S-Plus and Excel. This phase is quite time consuming. The chapters of the thesis are written in accordance with the results of the analyses.

Chapter I introduces the thesis. It reevaluates previous studies and cites the general results of these studies. There are explanations about the CDB90FT data set in this chapter. The scope of the thesis and some questions that the thesis answers are clearly indicated in the chapter.

In Chapter II, the background of the thesis is explained. In addition, definitions are provided that clarify the concepts in the data set. The main point of the chapter is to give general information about the history of military data analysis and to present the hierarchy of combat.

Chapter III focuses mainly on the concept of the Force Ratio (FR), which is accepted as one of the important predictors of battle outcome. The primary objective of the chapter is to figure out whether there are relationships between the FR and the battle outcome. The natures of the relationships, if they exist, are also examined.

In Chapter IV, the concept of dispersion is examined with respect to general trends through history. This chapter deals with the general changes in dispersion of troops in the battlefield, rather than relationships between dispersion and battle outcomes.

The last chapter, Chapter V, deals with the concept of the Daily Casualty Rate (DCR) and examines the relationships between DCR and unit size, as well as the general trends in DCR's.

The data analysis phase examines statements of general principles and their empirical relationships. We derive the logical consequences from the general principles. Some testing of hypotheses is conducted to evaluate the validity of the results. Finally, we compare our findings to previous studies.

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II. BACKGROUND

Military history is one of the oldest forms of historical scholarship. In the 19th century, the analysis and recording of military history was already highly developed and studied with a view to its lessons for the present and future. As Miksche, one of the original military thinkers of recent times, truly observed, "World history could not be understood if historians were to leave out the wars [Ref 2.1]."

Students of military art and military science have searched for fundamental laws or theories that would explain the interactions of military forces in combat and the outcomes of battles. The annals of military experience served as a guide for those students to get answers to the questions about the basic laws of combat.

The phenomenon of war has continued to fascinate scholars, particularly those trained in military skills. Those scholars tried to gain insights into the concept of warfare and to identify the patterns of war. Even when the results of a battle appear to have deviated from an identified pattern, an analyst generally reveals that this deviation is due to the operation of another pattern upon the circumstances of the battle.

A. THE HISTORY OF THE DATA COLLECTION ABOUT ANCIENT BATTLES

Since the oldest surviving military treatise, *"The Art of War,"* which was written by Sun Tzu in China about 500 BC [Ref 2.2], theorists of military science have sought fundamental laws or theories that would explain the interactions of military forces in combat and the outcomes of battles. Jomini, Clausewitz, Fuller, and Lanchester are prominent among the many theorists and scholars of military science who have studied historical battles.

Over the next 2,500 years, other thoughtful writers [Ref 2.2] on military affairs tried to formulate a theoretical approach to warfare. For instance, in the first century A.D. Sextus Julius Frontinus wrote books called *"On Military Affairs"* and *"Strategems."* About two centuries later another Roman, Flavirus Vegetius Renatus, wrote a book also titled *"On Military Affairs,"* more generally known as *"Military Institutions of Romans,"* which was a favorite reference work of ancient military scholars of Medieval Europe. Also, several theoretical works on war by Byzantines are worth mentioning: Mauricius's *"Strategikon," "The Tactica"* of Leo the Wise, and others.

In the century before Napoleon, there were such writings as *"Reveries on the Art of War"* by Count Maurice of Saxe, and *"Instructions to His Generals"* [Ref 2.2] by Frederick the Great of Prussia. Napoleon Bonaparte, as a commander, seemed to have formulated a theory on war in his own mind. While Napoleon hinted about this in his theoretical writings and in some of his correspondence, he demonstrated on the battlefield that there must be a theory of war. In his correspondence and recorded statements, as in *"Maxims,"* [Ref 2.2] he made it clear that his concepts on war had been derived basically from studying the campaigns of earlier great generals.

Henry Jomini [Ref 2.2], born in Switzerland in 1779, was an officer in Napoleon's army. Jomini tried to explain Napoleon's ideas on the theory of war, as he understood what Napoleon had in mind as clearly as anyone has. He also tried to model Napoleon's combat strategies and to understand the nature of the Napoleonic battlefield.

Carl von Clausewitz [Ref 2.2], a contemporary of Jomini born in 1780, wrote about many aspects of war, but his logical analytical approach focused on two of these aspects:

a) The activities of war,

b) War's characteristics—violence, passion, human behaviors, politics etc.

His concept of the outcome of a battle as a ratio is:

$$\text{Outcome} = (N_r * V_r * Q_r) / (N_b * V_b * Q_b)$$

where b = indices of blue force,

r = indices of red force,

N = number of troops,

V = variable circumstances,

Q = quality of force.

Denis Hart Mahan was the first great American military theorist. He compiled his own version of maxims and rules he thought relevant to military theory in America, but he never tried to produce a theory.

Helmuth von Moltke [Ref 2.2], 1800–1891, was both an eminent historian and an eminent military thinker. He was also a superb organizer and director of combat. However, he did little to advance military theory *per se*, other than in unrelated, although perceptive comments such as addressing the need to combine the tactical defensive with the strategic offensive. Moltke's military philosophy and strategic approach have survived to inspire successive generations of soldiers.

Alfred Thayer Mahan [Ref 2.2], 1840–1914, was an American military theorist in the style of his father, Denis Hart Mahan. His focus was on naval warfare and theory. A profound and gifted thinker on military and naval affaires, he well understood the relevance of military history to the contemporary military problems of his time. He recognized and analytically employed principles to military theory. More specifically,

Mahan investigated the Seven Years' War and the Napoleonic Wars. From this analysis, he wrote two classics: *The Influence of Sea Power upon History* and *The Influence of Sea Power upon the French Revolution and Empire, 1793-1812.* In 1890, with the publication of the first book, Mahan became the preeminent sea power historian and a classical theorist of naval warfare.

Count Alfred von Schilieffen [Ref 2.2] was the successor of Moltke as the Chief of the German General Staff. He was another thoughtful thinker on war who never attempted to distill a theory of combat from his encyclopedic knowledge of military history and the warfare of his time. The Schlieffen Plan once hailed has lately been questioned, and consequently it has inhibited any serious studies of his military genius. The plan that is named after him is referenced in the later chapters due to its effects on the conduct of the war.

John F. C. Fuller [Ref 2.2], the greatest military thinker of the 20th century, served in the British army and made his study of military history, particularly the campaigns of Napoleon, seeking insights on the basic principles of war. In 1921, he wrote *The Principles of War* proposing the following fundamentals of battle:

- objective,
- mass,
- offensive,
- surprise,
- security,
- movement,
- economy of forces,

- cooperation.

Frederick W. Lanchester [Ref 2.2], a contemporary of Fuller, based his ideas upon an analytical reading of military history. His primary concern was with the relationship between numerical strength and fighting strength. Most modern applications of Lanchester's work deal essentially with battlefield attrition. Lanchester formulated, using historical examples, a concept of warfare that could be expressed in two different equations:

Linear law : $dr/dt = \alpha * r * b$ (unaimed fire)

$db/dt = \beta * r * b$

Square law : $dr/dt = \alpha * b$ (aimed fire)

$db/dt = \beta * r$

where b = blue force level,

r = red force level,

α and β are attrition coefficients.

After examining many previous contributions to military data collection and the theory of war, we feel confident about the validity of this historic research. Therefore, we examine the theories of these great minds with respect to the results emerging from the CDB90FT data set.

B. THE IMPORTANCE OF DATA COLLECTION ABOUT ANCIENT BATTLES

The study of past experience is more vital in a world where military options are likely to be used with increasing precision and where the increasingly sensitive and well-informed societies will be less tolerant of mistakes.

What kind of analysis should be used?

Since World War II huge advances in mathematical and computer modeling of conflict have taken place. Within history, there has been a move towards increasingly minute analysis of short periods and limited areas. Historians may sometimes be criticized for extrapolating too readily and freely from the past to the present and to the future. The historian's answer to these critics is [Ref 2.3]: "What is your database?" Historians may have to be selective, but they can draw on the experience of everything that humanity has achieved, and everything that the human mind can comprehend.

The patterns of history are clear. While some influence of chance and factors cannot be explained clearly on the battlefield, these influences generally affect both sides. When the results of a battle appear to have deviated from an identified pattern, an analysis will usually reveal that this deviation was due to the operation of another pattern upon the circumstances of the battle.

The value of military history is that, when analyzed objectively and scientifically, it permits us to project the trends of real past experiences. This is one way that relevant lessons of actual combat can be learned. Understanding the inherent variation around the patterns is also important.

C. THE HIERARCHY OF COMBAT AND DEFINITIONS OF WAR, CAMPAIGN, BATTLE, ENGAGEMENT, ACTION, AND DUEL

In commonly accepted military terminology, there is a hierarchy of military combat, with war as its highest level, followed by *campaign, battle, engagement, action,* and *duel.*

A *war* [Ref 2.4] is an armed conflict, or a state of belligerence, involving military combat between two factions, states, nations, or coalitions. Hostilities between the opponents may be initiated with or without a formal declaration by one or both parties that a state of war exists. A war is fought for particular political or economic purposes or to resist an enemy's efforts to impose domination.

A *campaign* [Ref 2.4] is a phase of a war involving a series of operations related in time and space and aimed toward achieving a single, specific, strategic objective or result in the war. A campaign may include a single battle, but more often it comprises a number of battles over a protracted period of time or a considerable distance, but within a single theater of operations.

A *battle* [Ref 2.4] is combat between major forces, each having opposing operational missions, in which each side seeks to impose its will on the opponent by accomplishing its own mission, while preventing the opponent from achieving its objective. A battle starts when one side initiates combat, and it ends when one side accomplishes its mission or when one or both sides fail to accomplish the mission. Battles are often parts of campaigns.

An *engagement* [Ref 2.4] is combat between two forces, neither larger than a division nor smaller than a company, in which each side has an assigned mission. An engagement begins when the attacking force initiates combat in pursuit of its mission and ends when the attacker has accomplished the mission, or ceases to try to accomplish the mission, or when one or both sides receive significant reinforcements, thus initiating a new engagement. An engagement is often part of a battle.

An *action* [Ref 2.4] is combat between two forces, neither larger than a battalion nor smaller than a squad, in which each side has a tactical objective. An action begins when the attacking force initiates combat to gain its objective, and ends when the attacker wins the objective, or one or both forces withdraw, or both forces terminate combat. An action is often part of an engagement and sometimes is part of a battle.

A *duel* [Ref 2.4] is combat between two individuals or between two mobile fighting units. A duel begins when one side opens fire and ends when one side or both are unable to continue firing, or they stop firing voluntarily. A duel is frequently part of an action. Understanding the definitions related to combat, one can easily recognize that the CDB90FT data set is actually a set of battles. Thus, the conclusions, which are derived out of analyzes, are primarily about the battle level of the combat.

III. FORCE RATIO AND THE 3-TO-1 RULE

A. INTRODUCTION

In this chapter, the concept of Force Ratio (FR) (see below) is analyzed for the campaign-wise groupings of the CDB90FT data set. The analysis seeks general trends concerning FR in the campaigns and formulates the relations between trends and battle outcomes.

The data set is divided into 17 campaigns that comprise 552 battles out of the 660 battles in the actual data set. Campaigns that have less than six battles do not give consistent trends and were eliminated. The Arab-Israel War in 1948 was also eliminated due to a lack of data concerning force strengths.

The meaning of FR is examined from two aspects [Ref 3.1], namely validity and consistency, by analyzing each campaign. There are quite a lot of questions about the validity and the consistency of FR as a determiner of combat outcome. Despite some special conditions concerning the way the battles were conducted, many battles in history are presented as a counter argument against the validity and the consistency of FR. Thus, the results in this chapter are hypothesized and tested against counter arguments. However, the hypothesis testing in Logistics regression [Ref 3.2] is executed for the campaigns that include more than ten battles having FR values bigger than one so that the hypothesis testing is consistent over the number of battles. Otherwise, campaigns that have a small number of battles, within the limit mentioned above may complicate the overall results of the FR. That is, the sample sizes are too small to yield reliable estimates.

One of the main points of interest in this chapter is to determine the relationship, if any, between the FR and battle outcomes. Knowing that the attacker is the dominant power when the FR is higher than one, attackers' win/lose results are taken into account and correlated with the attackers' FR values. The results concerning the defender, on the other hand, are the compliments of the attackers' results.

B. THE CONCEPT OF FORCE RATIO (FR)

As a gross measure for campaign planning, the FR is useful and stands up quite well under historical scrutiny. As a basis for forecasting battle outcomes, however, it seems to be more probabilistic than deterministic. As such, the FR is less reliable in terms of predicting the battle outcome. The simple formula for FR is:

$$FR = A / D \tag{3.1}$$

where A is the total force strength of the attacker in manpower,

D is the total force strength of the defender in manpower.

The most well known form of FR is the 3-to-1 thumb rule [Ref 2.2] for the attacker. That the attacker should have 3-to-1 superiority over the defender before attacking is widely accepted as a guide in battle planning. Many commanders in history preferred to take defensive actions since their armies did not have numerical superiority over the other side. Actually, it is important to evaluate the force sizes before any battle. However, the conditions of the battlefield as a whole should be anticipated prior to any course of action.

Actually, the FR is a way to describe a battle. The battle may be a real one from history or a paper one from simulations. In both cases, the FR is used to predict the outcome of combat. The meaning of FR can be examined from two aspects: validity and

consistency. The first is the validity of the concept [Ref 3.1], that is, its ability to describe a battle as it actually occured. The second aspect is the ratio's consistency, that is, its ability to describe combat in a logical manner, without inconsistencies or contradictions.

In Operations Research (OR), Force Ratio (FR) is widely used in two areas [Ref 3.1]: predicting battle outcomes and estimating movement rates in combat. In making such predictions, an important point may be that a higher FR means a bigger probability of success. An insight into this assumption may be found in historical combat.

Use of the FR as a predictor of battle outcome implies acceptance of two assumptions. The first [Ref 3.1] is that only forces on the battlefield influence the outcome of the battle. Some analysts who are critical about this assumption mention that it may be the case in a war-gaming or a weapons study, but it has not always been so in real life. Some aspects other than the combatants in the battlefield have relative effects on the battle outcome. A commander in charge of defense, for example, may shift the correct position of his reserves due to changes in the attackers' formations. This mistake may cause the collapse of the defense line.

The second assumption [Ref 3.1] is that troops of each side are assumed to be identical in terms of discipline, personal strength etc. Although this may be questionable, it does not distort our results. The analyst of data tries to determine trends among many numbers. In fact, the analyst may not be so much interested in the qualitative features of the data set, such as discipline of forces, training quality of troops etc. while executing the analysis, since the numerical features define the relationships among variables. In his respect, the assumption is not a vital one to accept for analyses. However, starting from the point that only the same kind of elements can be compared to each other, apples-to-

apples or oranges-to-oranges, accepting this assumption becomes logical. Having said that, it is time to analyze campaigns chronologically from ancient to modern, so that trends can be traced sequentially.

C. THIRTY YEARS' WAR—(1620-1648)

1. History of the Campaign

One of the most savage conflicts in history, The Thirty Years' War [Ref 3.3] grew from a revolt in Bohemia into a European struggle between Catholic and Protestant powers. The war then developed into a political struggle against the house of Hapsburg, first by Sweden and then France. Battles were fought on German soil. The Treaties of Westphalia, signed in 1648, which granted indemnities to Sweden and France, recognized the republics of Netherlands and Switzerland, and provided more religious toleration for Protestants, ended the campaign. Germany lay prostrate, deprived of manpower and wealth.

2. Analysis of Force Ratio

Since there are not great rational differences between attacker and defender forces in this campaign, no battle is listed in [Table 3.1] cases of FR = 3 or more, FR = 3-2.5, and FR = 2.5-2. Also, only one battle that ended in the attacker's success is mentioned in the FR = 2-1.5 case. It is quite interesting to see that four battles in which the attackers had a slight numerical superiority over the defenders, for instance FR = 1.3-1, ended in the attackers' success on the battlefield. Win cases in the CDB90FT data set that are used throughout the thesis are subjective and sometimes debatable.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)				1			4	13
Number of Battles Attacker Wins (a)				1			4	12
P(Attacker wins given FR)				**1**			**1**	**0.923**

Table 3.1. P(Attacker wins given FR) Values Corresponding to Each FR Category.

When the overall picture of the campaign is examined, some battles stand differently in one way or another. In the battle of Dessau Bridge in 1626 [Table 3.2], the attacking Union army was almost half the size of the defending Imperial army having a battle FR = 0.53: 8500 and 16000 respectively. It is also worth mentioning that the attackers won 7 out of 11 battles, in which the FR values are less than one. The total troops in the battle, on the other hand, stayed well below 50,000, except the battle of Alte Veste, when the defending Imperial army gathered 60,000 troops on the battlefield.

Battle Name	**Year**	**Total Strength of the attacker A**	**Total Strength of the defender D**	**Win case {1 if A, -1 if D, 0 o.w.}**	**Force Ratio(FR) A / D**
WHITE MOUNTAIN	1620	40000	21000	1	**1.905**
WIMPFEN	1622	20000	20000	1	**1.000**
DESSAU BRIDGE	1626	8500	16000	-1	**0.531**
LUTTER	1626	17000	20000	1	**0.850**
BREITENFELD I	1631	32000	37850	-1	**0.845**
THE LECH	1632	33000	27000	1	**1.222**
ALTE VESTE	1632	46000	60000	-1	**0.767**
LUETZEN	1632	18996	21770	1	**0.873**
NORDLINGEN I	1634	25000	35000	-1	**0.714**
WITTSTOCK	1636	22000	30000	1	**0.733**
BREITENFELD II	1642	25000	30000	1	**0.833**
ROCROI	1643	23000	26000	1	**0.885**
TUTTLINGEN	1643	22000	18000	1	**1.222**
FREIBURG	1644	19000	16000	1	**1.188**
JANKAU	1645	15000	15000	-1	**1.000**
MERGENTHEIM	1645	10000	11000	1	**0.909**
ALLERHEIM	1645	18000	16000	1	**1.125**
LENS	1648	14000	18000	1	**0.778**

Table 3.2. FR Value of Each Battle.

D. ENGLISH CIVIL WAR—(1642-1645)

1. History of the Campaign

Increasing friction between the English crown and Parliament finally flared into open warfare [Ref 3.3] in 1642. In the ensuing civil war, sometimes called the "Great Rebellion," King Charles I was supported by the Anglican episcopacy, while the Presbyterians and other reformers took the side of Parliament. The Parliament's forces defeated the King and the monarchy was abolished, which transferred the power of the republic to the protectorate of General Cromwell.

2. Analysis of Force Ratio

There are six battles listed in this campaign and none of them has a force ratio that is bigger than 2.5, thus, leaving FR = 3 or more [Table 3.3] and FR = 3-2.5 [Table 3.3] cases not available (NA). On the other hand, the defenders won two battles, namely the battle of Tippermuir in 1644 and Newbury in 1644 [Table 3.3], in which the attackers have a FR = 2-1.5 superiority over the defenders while the battle of Marston in 1644 with a FR=1.54 [Table 3.3] was won by the attacker.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)			2	1				3
Number of Battles Attacker Wins (a)			0	1				2
P(Attacker wins given FR)			**0**	**1**				**0.667**

Table 3.3. P(Attacker wins given FR) Values Corresponding to Each FR Category.

Meanwhile, obviously the sizes of the units in the battles during the campaign changed greatly. In the battle of Marston [Table 3.4], for example, the attacking English Parliamentary army had 27,000 troops in the battlefield while the Scot army of 3,000 defended against the attacker in the battle of Tippermuir. Since there are not so many battles in the campaign, each battle has relatively dramatic effects over all numerical

features of the campaign making the results of analyses special to the campaign itself. Thus, the results concerning the battles in the campaign are rather more local than general for all the campaigns listed in the data set.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A/D
EDGEHILL	1642	14300	14870	1	0.962
MARSTON	1644	27000	17500	1	1.543
TIPPERMUIR	1644	6800	3000	-1	2.267
KILSYTH	1644	4900	6800	1	0.721
NEWBURY II	1644	22000	10000	-1	2.200
NASEBY	1645	9000	13000	-1	0.692

Table 3.4. FR Value of Each Battle.

E. KING WILLIAM'S WAR—(1689-1693)

1. History of the Campaign

Hostility between the New England colonists and the French and Indians [Ref 3.3] became a formal conflict when England declared war on France in 1689. The only major battle in North America was fought at Port Royal in Nova Scotia in 1690. Named for King William III of Great Britain, the colonial war ended with the 1697 Peace of Ryswick.

2. Analysis of Force Ratio

Close numbers of attacker and defender forces in the campaign made FR values relatively small leading to NA for FR = 3 or more [Table 3.5], FR = 3-2.5 [Table 3.5], and FR = 2.5-2 [Table 3.5] cases. Two battles in FR = 2-1.5 case [Table 3.5] with FR values of 1.52 and 1.60 ended with attackers' success. Three other battles are examined

in the campaign in terms of the FR values; the attackers won two of them, except for the battle of Steenkerke in 1692 in which the defending French army defeated the attacking Allied force: FR = 1.05.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)				2		1	2	3
Number of Battles Attacker Wins (a)				2		1	1	2
P(Attacker wins given FR)				**1**		**1**	**0.5**	**0.667**

Table 3.5. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The unit sizes in the campaign varied widely between 2,800 and 80,000. In the battle of Killiecrankie [Table 3.6] in 1689, the attacking Scot army of 2,800 defeated the defending English army of 3,400. Thus, this battle is relatively smaller than the battle of Neerwinden [Table 3.6] in 1693, in which the French army of 80,000 attacked the Allied army of 50,000 in terms of the unit sizes. It is also worth mentioning that the attackers were successful in six battles out of eight in the campaign.

Battle Name	**Time**	**Total Strength of the attacker A**	**Total Strength of the defender D**	**Win case {1 if A, -1 if D, 0 o.w.}**	**Force Ratio(FR) A / D**
KILLIECRANKIE	1689	2800	3400	1	**0.824**
WALCOURT	1689	24000	35000	-1	**0.686**
FLEURUS	1690	50000	38000	1	**1.316**
THE BOYNE	1690	35000	23000	1	**1.522**
AUGHRIM	1691	18000	25000	1	**0.720**
STEENKERKE	1692	63000	57000	-1	**1.105**
NEERWINDEN	1693	80000	50000	1	**1.600**
MARSAGLIA	1693	40000	36000	1	**1.111**

Table 3.6. FR Value of Each Battle.

F. AUSTRIAN SUCCESSION WAR—(1741-1745)

1. History of the Campaign

When Maria Theresa succeeded her father, Charles VI, as Holy Roman Emperor [Ref 3.3], her right to reign was challenged by Bavaria, Spain, Poland, and Saxony. Prussia claimed the province of Silesia and seized the disputed province in 1740. This conflict then merged into the larger war of the Austrian Succession. France, Spain, and Bavaria allied themselves with Prussia, while Great Britain, the Netherlands, and Sardinia supported Maria Theresa and Austria.

Finally in 1748 the Peace of Aachen ended the war. Most of the conquered territory was restored, except for Silesia, which passed to Prussia. Maria Theresa's rule was guaranteed, and her husband won recognition as Holy Roman Emperor Francis I.

2. Analysis of Force Ratio

The peculiarity of this campaign is that no battle was fought with an FR of 1.5 or more so that all those values of the FR tables are NA. Although the campaign can be held as an outlier and extracted from the analysis due to an insufficient number of entries, a probable loss of generality prevents doing so. Notice that battles with the highest value of FR = 1.34 [Table 3.7] and the lowest FR = 0.55 [Table 3.7] ended in zero for both sides.

When the FR values that are less than 1.4 are examined, two battles fall in the limit of FR = 1 or more. In the battle of Dettingen [Table 3.8] in 1743, the attacking French army had a numerical superiority over the defending British army by the FR = 1.346 while the Austrian army of 29,000 had a numerical superiority over the Prussian army of 24,500 in the battle of Chotusits in 1742: FR = 1.184. The first battle was won by the attacker as the defender succeeded in the second one.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)						1	1	5
Number of Battles Attacker Wins (a)						1	0	4
P(Attacker wins given FR)						**1**	**0**	**0.8**

Table 3.7. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The campaign was virtually a tie in terms of the outcomes of the battles for the defender and the attacker. The attacker succeeded in four battles out of seven while the remaining three ended in the success of the defenders. Meanwhile, we should note that the attackers had numerical superiority over the defender in two battles, namely the battle of Chotisitz and the battle of Dettingen [Table 3.8]. The rest of the battles, on the other hand, were conducted under the defender's numerical superiority.

The FR values varied widely between 1.346 and 0.5503 [Table 3.8]. The attackers, however, claimed victory over the defenders in three out of five cases in which the defenders enjoyed a numerical superiority. It is quite interesting that the attacking Prussian army defeated the defending Austrian army in the battle of Soor in which the attacker was half the size of the defender: FR = 0.5503.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
MOLLWITZ	1741	18100	22000	-1	**0.823**
CHOTUSITZ	1742	29000	24500	-1	**1.184**
DETTINGEN	1743	35000	26000	1	**1.346**
FONTENOY	1745	50000	60000	-1	**0.833**
HOHENFRIED	1745	50000	66000	1	**0.758**
SOOR	1745	22562	41000	1	**0.550**
KESSELDORF	1745	31000	31200	1	**0.994**

Table 3.8. FR Value of Each Battle.

G. SEVEN YEARS' WAR—(1756-1760)

1. History of the Campaign

The undeclared war between France and Great Britain that began in North America in 1754 helped provoke a general conflict [Ref 3.3] in Europe two years later. Called the Seven Years' War, it found Austria, Russia, France, Sweden, and Poland aligned against Prussia, Great Britain, and Portugal. The Treaty of Paris in 1763 ended the Seven Years' War as well as the French and Indian War. Great Britain received Minorca, Canada, and Florida. France ceded Louisiana to Spain; Prussia retained Silesia.

2. Analysis of Force Ratio

It is noteworthy that this campaign is the very first group of battles that has battles with high FR values. The battle of Hastenbeck in 1757 was fought between attacking French forces and defending Prussian forces with FR = 16.82 [Table 3.9] showing that the attacking force is 16.82 times higher than the defending one. Despite the huge superiority of the attacker, the defending Prussian forces defeated the French. For five other cases, P(A wins given FR) stays between 0.5 [Table 3.9] and 1 [Table 3.9].

The battle of Hastenbeck [Table 3.10] in 1757 might be accepted as an outlier since the conduct of the battle itself differed from the other cases. In this specific battle, the disciplinary superiority of Prussian army proved to be the determining factor of the battle.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	1	2		2			1	12
Number of Battles Attacker Wins (a)	0	2		1			1	7
P(Attacker wins given FR)	**0**	**1**		**0.5**			**1**	**0.583**

Table 3.9. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The campaign of the Seven Years' War was completely dominated by the Prussian and British armies' superiority in either discipline or the conduct of the war. Thus, the Prussian army mostly determined the outcomes of the battles. In the battle of Plassey [Table 3.10] in 1757, the defending British army of 2,975 defeated the Bengali force of 50,050, which was almost 17 times higher than the British army. Also, in the battle of Leuthen, the attacking Prussian army defeated the defending Austrian force which is two times higher than the attacking army.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
LOBOSITZ	1756	29000	34500	1	**0.841**
PRAGUE	1757	65000	62000	1	**1.048**
PLASSEY	1757	50050	2975	-1	**16.824**
KOLIN	1757	32000	44000	-1	**0.727**
HASTENBECK	1757	60000	36000	1	**1.667**
ROSSBACH	1757	42000	22000	-1	**1.909**
LEUTHEN	1757	33000	65000	1	**0.508**
CREFELD	1758	32000	50000	1	**0.640**
ZORNDORF	1758	36000	43300	1	**0.831**
HOCHKIRCH	1758	80000	31000	1	**2.581**
BERGEN	1759	24000	30000	-1	**0.800**
MINDEN	1759	45000	60000	1	**0.750**
KUNERSDORF	1759	50900	59500	-1	**0.855**
PLAINS OF ABR	1759	4500	4800	-1	**0.938**
MAXEN	1759	38000	13500	1	**2.815**
WARBURG	1760	19000	17000	1	**1.118**
LIEGNITZ	1760	30000	30000	-1	**1.000**
TORGAU	1760	50000	53400	1	**0.936**

Table 3.10. FR Value of Each Battle.

H. AMERICAN REVOLUTIONARY WAR—(1775–1781)

1. History of the Campaign

A strong American resentment [Ref 3.3] against British rule developed after the successful conclusion of the French and Indian War. Widening the gap between the 13 colonies and the mother country were the Stamp Act (1765), the Boston Massacre (1770), the Boston Tea Party (1773), and the Intolerable Act (1774).

The first pitched battles between colonial militia and British regulars took place at Lexington and Concord, both in Massachusetts, on April 19, 1775. On July 4, 1776, American patriots announced their Declaration of Independence. This historic act, together with the decisive U.S. victory at Saratoga in 1777, gained the allegiance of France.

The Treaty of Versailles ended the war. The independence of the United States was acknowledged, conquests in India were mutually restored, and Florida and Minorca ceded to Spain.

2. Analysis of Force Ratio

Five of the battles are in analyses' tables of FR: the battle of Trenton with FR = 1.59 [Table 3.11], the battle of Princeton with FR = 4 [Table 3.11], the battle of White Plains with FR = 1.0, the battle of Germantown with FR = 1.244, the battle of Monmouth with FR = 1.182, the battle of Cowpens with FR = 1.073, the battle of Eutaw Springs with FR = 1.10. Just three of battles ended in the success of the attackers. A look at the FR values reveals that battles in this campaign were fought between almost equal forces making FR values close to one.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	1			1			3	9
Number of Battles Attacker Wins (a)	1			1			0	5
P(Attacker wins given FR)	**1**			**1**			**0**	**0.556**

Table 3.11. P(Attacker wins given FR) Values Corresponding to Each FR Category.

As the unit sizes of the battles are examined through the campaign, it is well understood that the sizes of the units [Table 3.12] remained relatively small ranging between 900 and 13,000. There were vast shortages of troops in the battlefield. The American army faced severe problems supporting the battlefield with enough troops. The British army faced the same problems due to their distance from the motherland.

Battle Name	Year	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
BUNKER HILL	1775	2650	3200	1	**0.828**
QUEBEC	1776	1100	1800	-1	**0.611**
WHITE PLAINS	1776	13000	13000	1	**1.000**
TRENTON	1776	2420	1520	1	**1.592**
PRINCETON	1777	4800	1200	1	**4.000**
FREEMAN'S FARM	1777	4400	7000	0	**0.629**
GERMANTOWN	1777	11200	9000	-1	**1.244**
BEMIS HEIGHTS	1777	5000	11000	-1	**0.455**
MONMOUTH	1778	13000	11000	0	**1.182**
CAMDEN	1780	2100	3050	1	**0.689**
COWPENS	1781	1100	1025	-1	**1.073**
GUILFORD COURT	1781	1900	4449	1	**0.427**
HOBKIRK'S HILL	1781	900	1551	1	**0.580**
EUTAW SPRINGS	1781	2200	2000	-1	**1.100**

Table 3.12. FR Value of Each Battle.

I. WAR OF FIRST COALITION—(1792-1799)

1. History of the Campaign

The Legislative Assembly, which had assumed the Revolutionary power in France in 1791, became the target of a Prussian-Austrian alliance the following February [Ref 3.3]. These powers were joined by Sardinia, Great Britain, the Netherlands, Spain, Naples, and the Papal States to form the First Coalition. The war opened in 1792 with the French victory over Prussia at Valmy. Under the National Convention, France drove first Prussia and then Spain out of the war. The Piedmont, Naples, and the Papal States were overrun.

2. Analysis of Force Ratio

The battle of Valmy, which is mentioned in the history section, is the very first battle of the data set [Table 3.7]. Despite being outnumbered by attacking Coalition Forces, the defending French army claimed a clear victory.

The first two of three battles that have FR values of 3.07, 3.23, and 7.778 were won by attacking forces, while the battle of Mount Tabor in Palestine, in which the attacking forces had 7.7778-to-1 superiority over the defending forces [Table 3.13] ended in the victory of the defending forces. Also, worth mentioning is that two more battles with FR values of 1.91 and 1.7 were dominated by attackers' victories [Table 3.13]. Six more battles were examined in the cases of FR<=1. The defenders succeeded in only two battles out of six, namely the battle of Rivoli in 1798 and the battle of Neerwinden in 1793.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	3			2	1	2	2	4
Number of Battles Attacker Wins (a)	2			2	1	1	2	1
P(Attacker wins given FR)	**0.667**			**1**	**1**	**0.5**	**1**	**0.25**

Table 3.13. P(Attacker wins given FR) Values Corresponding to Each FR Category.

This campaign is almost a perfect example of the effectiveness of the FR in the battlefield. When the FR value for the battle is higher, for example FR = 7.778 of the battle of Mounth Tabor [Table 3.14], the win probability of the attacker becomes greater. On the contrary, the attacker is more likely to lose the battle if the FR value of the battle becomes smaller, as in the battle of Fleurus with FR = 0.630. Also, it is interesting that 10 out of 14 battles were fought under the numerical superiority of the attackers.

Battle Name	**Time**	**Total Strength of the attacker A**	**Total Strength of the defender D**	**Win case {1 if A, -1 if D, 0 o.w.}**	**Force Ratio (FR) A / D**
VALMY	1792	34000	36000	-1	**0.944**
JEMAPPES	1792	40000	13000	1	**3.077**
NEERWINDEN	1793	45000	43000	-1	**1.047**
HONDSCHOOTE	1793	42000	13000	1	**3.231**
WATTIGNIES	1793	44000	23000	1	**1.913**
FLEURUS	1794	46000	73000	-1	**0.630**
LODI	1796	17000	10000	1	**1.700**
CASTIGLIONE	1796	30000	25000	1	**1.200**
NERESHEIM	1796	40000	45000	1	**0.889**
WUERZBURG	1796	44000	30000	1	**1.467**
ARCOLA	1796	17300	12700	1	**1.362**
RIVOLI	1797	28000	20500	-1	**1.366**
PYRAMIDS	1798	25000	21000	1	**1.190**
MOUNT TABOR	1799	35000	4500	-1	**7.778**

Table 3.14. FR Value of Each Battle.

J. WAR OF SECOND COALITION—1800

1. History of the Campaign

It was actually the second phase [Ref 3.3] of the War of the First Coalition. When Austria, Naples, Portugal, and the Ottoman Empire entered the conflict, the War of the Second Coalition took shape. Russia withdrew in 1799. This phase ended when Austria, Naples, and Great Britain made peace with France. The French stood supreme on land.

2. Analysis of Force Ratio

All of the battles in this campaign, excluding three with FR values of 1.6, 1.429, and 1.52 [Table 3.15], were fought between almost equal forces. However, the numerical superiority of the attackers continued in six out of seven battles, but slightly. Although it seems that both defender and attacker sides succeeded once in two cases, having FR values of 1.6 and 1.52, the same Allied Force defeated the French in both battles.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)				2	1			4
Number of Battles Attacker Wins (a)				1	1			1
P(Attacker wins given FR)				**0.5**	**1**			**0.25**

Table 3.15. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The defenders in the campaign dominated over the attackers in four battles out of seven [Table 3.16]. Since the FR values during the campaign remained close to one, the defenders had the chance of using the ground superiority or the superiority in positioning to suppress the attackers so that they were more likely to succeed in the battles than the attackers. Also important, the defender almost always has the chance of determining the front line on which the battle is fought. This favors the defender to impose its will on the attacker if the FR value is in the close vicinity of one or less.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
STOCKACH I	1799	38000	50000	-1	**0.760**
ZURICH I	1799	40000	25000	-1	**1.600**
NOVI	1799	50000	35000	1	**1.429**
ZURICH III	1799	35000	23000	1	**1.522**
MOESKIRCH	1800	60000	60000	1	**1.000**
MARENGO	1800	31000	29000	-1	**1.069**
HOHENLINDEN	1800	57000	55000	-1	**1.036**

Table 3.16. FR Value of Each Battle.

K. NAPOLEONIC WARS—(1805-1815)

1. History of the Campaign

During the wars of the French Revolution, France held its own against both the First and the Second Coalitions of European powers. Napoleon Bonaparte, who had risen to first consul in 1799 and then to emperor of France in 1804, ruled an empire already embroiled in a new conflict [Ref 3.3]. Great Britain, which had resumed the war in 1803, was joined by Austria, Russia, Sweden, and Naples. In this War of the Third Coalition, Spain sided with France. Austria was knocked out of the war in 1805, and the Holy Roman Empire was dissolved the following year.

2. Analysis of Force Ratio

This campaign is one of the important groups of the battles since the number of the battles in the campaign is quite large for deriving a more consistent trend about the concept of the FR. Out of 29 battles in the Napoleonic Wars, eight battles [Table 3.17] have FR values higher than 1.5. Eleven more battles are examined in the cases that have the FR<1.3. Actually, the Napoleonic Wars are the first group of battles in history that can be named as a global conflict in terms of the nations involved and the territory. Some special conditions in the campaign (see Chapter V) remained dominant in the battlefields,

such as the commanders' willingness to concentrate forces in the battlefield so that the operation would be successful. This intention of generals in charge of either the attacker or the defender caused an increase in the number of combatants in the battlefield.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)		2	2	4	1	2	8	10
Number of Battles Attacker Wins (a)		2	1	4	0	1	4	2
P(Attacker wins given FR)		**1**	**0.5**	**1**	**0**	**0.5**	**0.5**	**0.2**

Table 3.17. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The unit sizes remained relatively high during the campaign due to the conditions mentioned above. The smallest force is the French army of 13,050 that attacked the British army in the battle of Vimiero [Table 3.18] in 1808. The biggest force, on the other hand, is the Allied force of 365,000 that attacked the defending French army in Leipzig in 1813. Also, note that the unit sizes varied widely between 6-digit and 5-digit numbers [Table 3.18]. The campaign itself was fought among the nations of Europe that were capable of supporting the battlefield with an ample amount of troops. This factor is the other effective aspect governing the increase in the number of troops in the battlefields.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
AUSTERLITZ	1805	85400	73200	-1	**1.167**
JENA	1806	96000	53000	1	**1.811**
AUERSTADT	1806	63500	27000	-1	**2.352**
EYLAU	1807	78000	80000	0	**0.975**
FRIEDLAND	1807	80000	60000	1	**1.333**
VIMIERO	1808	13050	19600	-1	**0.666**
CORUNNA	1809	20600	14800	-1	**1.392**
ECKMUEHL	1809	74000	66000	-1	**1.121**
ASPERN-ES	1809	99000	66000	1	**1.500**
THE RAAB	1809	35000	37000	1	**0.946**
WAGRAM	1809	140000	140000	1	**1.000**
TALAVERA	1809	46000	54500	-1	**0.844**
BUSSACO	1810	65900	51910	-1	**1.270**
FUENTES	1811	48260	37360	-1	**1.292**
ALBUERA	1811	23000	30000	-1	**0.767**
SALAMANCA	1812	46000	42000	1	**1.095**
VITTORIA	1813	79062	68024	1	**1.162**
BORODINO	1812	120000	120000	1	**1.000**
LUETZEN	1813	93000	120000	-1	**0.775**
BAUTZEN	1813	199000	97000	1	**2.052**
DRESDEN	1813	170000	120000	-1	**1.417**
LEIPZIG	1813	365000	196200	1	**1.860**
HANAU	1813	60000	40000	1	**1.500**
LA ROTHIERE	1814	110000	40000	1	**2.750**
LAON	1814	47600	85000	-1	**0.560**
ARCIS-SUR	1814	80000	30000	1	**2.667**
LIGNY	1815	67567	82895	1	**0.815**
QUATRE BRAS	1815	26741	33765	-1	**0.792**
WATERLOO	1815	68265	137547	-1	**0.496**

Table 3.18. FR Value of Each Battle.

This campaign is the first of four campaigns for which the Logistics regression [Ref 3.2] is applied since it has 19 battles that have FR>=1. It is assumed that the outcomes of the battles have relationships with the FR in the Napoleonic War. The hypothesis testing for the model [Ref 3.6] is:

H_o: $\beta_1 = 0$ (There is no relationship between FR and P(Win))

H_a: $\beta_1 \neq 0$ (There is relationship between FR and P(Win)) (3.1)

where β_1 = the coefficient of FR in the Logistic Regression [Ref 3.6].

When the Logistics regression is executed for the campaign to evaluate whether Ho of the model (3.1) is valid or not, the following results from S-Plus are achieved:

0.85 is our estimated coefficient (β_1) associated with FR. A positive β_1 means that the probability of winning increases as FR increases.

```
(Intercept)      FR
 -0.7716269 0.8576075
```

To evaluate the p-value of the model, the residual deviance and the residual degrees of freedom are used:

```
1-pchisq(24.2313,17)
[1] 0.1132854
```

(3.2)

The p-value of 0.11 says that we would see data like this 11% of the time if there were no relationship. This suggests a relationship, however we would not reject H_o at the 0.05 significance level. We might see more definitive results with more than 19 data points.

L. US-MEXICO WAR—(1846-1847)

1. History of the Campaign

A dispute over the territory between the Nueces and Rio Grande rivers led to an open conflict [Ref 3.3] between the United States and Mexico in April 1846. On May 13, the U.S. declared war. The fighting lasted 17 months, ending with American troops in possession of California, the Southwest, northern Mexico, and central Mexico from Veracruz to Mexico City. By the Treaty of Guadalupe Hidalgo, Mexico ceded the territories of California and New Mexico and all the land extending to the Rio Grande.

2. Analysis of Force Ratio

One peculiarity of the campaign is that all the battles except for two of them with FR values of 2.94 and 1.12 [Table 3.19] were fought between numerically superior defenders and inferior attackers. Only the battle of Buena Vista with FR = 2.94 and the

battle of Contreras with FR = 1.125 show up as examples in which the attackers had numerical superiority over the defenders.

In the campaign, the American forces won all the battles. Even though the Mexican army defended in all battles, in the battle of Bueno Vista [Table 3.20] in 1847 the Mexican army was almost three times higher than the defending American forces. However, the battle ended in a clear victory for the American forces.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)		1					1	6
Number of Battles Attacker Wins (a)		0					1	6
P(Attacker wins given FR)		0					1	1

Table 3.19. P(Attacker wins given FR) Values Corresponding to Each FR Category.

Although the attackers succeeded in seven out of eight battles [Table 3.20], the American forces claimed victory in all battles, even in the battle that the Mexican army had a clear numerical superiority. These results were mainly due to the qualified leadership of the American forces and their superior firepower.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
PALO ALTO	1846	2288	6000	1	0.381
RESACA	1846	1700	5600	1	0.304
BUENA VISTA	1847	14000	4759	-1	2.942
CERRO GORDO	1847	8500	12000	1	0.708
CONTRERAS	1847	4500	4000	1	1.125
CHURUBUSCO	1847	8497	10500	1	0.809
MOLINO DEL REY	1847	3100	12000	1	0.258
CHAPULTEPEC	1847	7180	15000	1	0.479

Table 3.20. FR Value of Each Battle.

M. AMERICAN CIVIL WAR—(1861-1865)

1. History of the Campaign

The issue of slavery, particularly in the new states being formed from western territories, drove an ever-larger wedge [Ref 3.3] between the free states of the North and the slave-holding states in the South. When the Republican candidate for President of the United States, Abraham Lincoln, won election on November 6, 1860, the situation reached a crisis.

For four years the United States was torn by bitter civil war. The major theater of operations was east of the Appalachians, especially in northern Virginia between the two hostile capitals of Washington D.C., and Richmond. In the costliest war in the United States history, the Confederate government was decisively defeated, the Union was preserved, and slavery was abolished.

2. Analysis of Force Ratio

Since there are 49 battles listed in the data set for the campaign, this group of battles can be taken into account as one of the campaigns that bears general trends. Important to mention is that the attackers won all of the three battles that have FR values of 15.05, 5, and 3 [Table 3.21]. The very early hypothesis that the probability of winning for the attacker decreases as the FR decreases is revealed in the campaign. This hypothesis is tested in the Conclusions Section.

This campaign comprises one of the largest battle groups in terms of the battles that are listed in the data set. The number of battles [Table 3.22], 49, which is much larger than the thumb number of the normal distribution, 30, is sufficient to bear the general trends governing the FR analysis. The results that are derived from the campaign

are expected to dominate the overall results relative to the number of battles in the campaign.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	3		6	12	3	1	10	14
Number of Battles Attacker Wins (a)	3		3	3	2	1	2	2
P(Attacker wins given FR)	**1**		**0.5**	**0.25**	**0.667**	**1**	**0.2**	**0.143**

Table 3.21. P(Attacker wins given FR) Values Corresponding to Each FR Category.

Battle Name	Year	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
FIRST BULL RUN	1861	35000	32500	-1	**1.077**
WILSON'S CREEK	1861	5400	10175	-1	**0.531**
BELMONT	1861	3144	5000	1	**0.629**
MILL SPRINGS	1862	4000	4000	-1	**1.000**
FORT DONELSON	1862	21000	27000	-1	**0.778**
PEA RIDGE	1862	16202	10500	-1	**1.543**
KERNSTOWN	1862	3087	7000	-1	**0.441**
SHILOH	1862	40355	66812	-1	**0.604**
FRONT ROYAL	1862	16000	1063	1	**15.052**
FIRST WINCHESTER	1862	16000	7000	1	**2.286**
CROSS KEYS	1862	10500	5000	-1	**2.100**
PORT REPUBLIC	1862	15000	3000	1	**5.000**
SEVEN PINES	1862	41816	41797	-1	**1.000**
MECHANICSVILLE	1862	16808	15631	-1	**1.075**
GAINES'S MILL	1862	57018	34214	-1	**1.667**
GLENDALE	1862	86748	83345	-1	**1.041**
MALVERN HILL	1862	82507	78902	-1	**1.046**
CEDAR MOUNTAIN	1862	8030	16848	-1	**0.477**
SECOND BULL RUN	1862	75696	48527	-1	**1.560**
SOUTH MOUNTAIN	1862	28480	17852	1	**1.595**
ANTIETAM	1862	90000	46000	-1	**1.957**
CORINTH	1862	22000	21147	-1	**1.040**
PERRYVILLE	1862	36940	16000	0	**2.309**
FREDERICKSBURG	1862	106007	72497	-1	**1.462**
MURFREESBORO	1862	34732	41400	0	**0.839**
CHANCELLORSVILLE	1863	113000	60892	-1	**1.856**
CHAMPION'S HILL	1863	29373	20000	1	**1.469**
BRANDY STATION	1863	12000	10000	1	**1.200**
GETTYSBURG	1863	75054	83289	-1	**0.901**
CHICKAMAUGA	1863	66326	58222	1	**1.139**
CHATTANOOGA	1863	61000	40000	1	**1.525**
THE WILDERNESS	1864	101895	61025	-1	**1.670**
SPOTSYLVANIA	1864	90000	50000	-1	**1.800**
NEW MARKET	1864	5000	5150	1	**0.971**
COLD HARBOR	1864	107907	59000	-1	**1.829**
KENESAW MOUNTAIN	1864	16225	17733	-1	**0.915**
PEACHTREE CREEK	1864	18832	20139	-1	**0.935**
ATLANTA	1864	36934	30477	-1	**1.212**
PETERSBURG	1864	63797	41499	-1	**1.537**
GLOBE TAVERN	1864	20289	14787	1	**1.372**
OPEQUON CREEK	1864	37711	17103	1	**2.205**
CEDAR CREEK	1864	18410	30829	-1	**0.597**
FRANKLIN	1864	26897	27939	-1	**0.963**
NASHVILLE	1864	49773	23207	1	**2.145**
BENTONVILLE	1865	27000	60000	-1	**0.450**
DINWIDDIE	1865	45247	20030	0	**2.259**
FIVE FORKS	1865	30000	10000	1	**3.000**
SELMA	1865	13500	7000	1	**1.929**
SAYLOR'S CREEK	1865	30000	21000	1	**1.429**

Table 3.22. FR Value of Each Battle.

The attackers did not adhere to the absolute numerical superiority over the defenders, namely following the “three-to-one” rule of thumb. Rather, they preferred a relatively larger number of troops in the battlefield having the FR values [Table 3.22] between 1 and 2.25 for the cases when the FR was greater than one. That particular approach might be due to the combat experience of the battle commanders. Since the generals of both sides knew how their opponents conducted war, the attackers, either the Union or the Confederate, sought relative equality in the number of the numerical superiority making the FR values consistent with each other.

The hypothesis testing is conducted for the American Civil War since the war includes 35 battles that have FR values greater than one. The hypothesis testing for the model [Ref 3.6] is:

H_o: $\beta_1 = 0$ (There is no relationship between FR and P(Win))

H_a: $\beta_1 \neq 0$ (There is relationship between FR and P(Win)) (3.3)

To test the validity of the model (3.3), the Logistics regression is applied. 1.71 is our estimated coefficient (β_1) associated with FR. A positive β_1 means that the probability of winning increases as FR increases.

```
(Intercept)     FR
 -3.200175 1.718775
```

Actually, the model is tested against α=0.05. The p-value of the model is calculated and compared to α=0.05:

```
1-pchisq(37.51548,31)
[1] 0.1951384
```
(3.4)

The p-value of 0.19 says that we would see data like this 19% of the time if there were no relationship. This suggests a relationship, however we would not reject H_o at the 0.05 significance level.

N. FRANCO-PRUSSIAN WAR—(1870-1871)

1. History of the Campaign

The third, and the last, of the wars fought by Otto von Bismarck to forge a German empire was aimed at France [Ref 3.3]. The French emperor, Napoleon III, was provoked into a declaration of war on July 15, 1870. Six weeks later, the conflict was virtually ended by the overwhelming Prussian victory at Sedan.

2. Analysis of Force Ratio

Many of the battles in the campaign were fought under the attackers' numerical superiority [Table 3.23]. The disciplinary and numerical superiority of the Prussian army played a dominant role through the campaign, thus, enabling the Prussian army to be more likely to succeed in the battlefield. One noticeable point is that seven battles that have FR>1 were won by the attackers while the battle of Belfort in which the German army battled the numerically superior French army ended in the clear victory of the Prussian army over the French.

The Prussian army was successful in nine out of ten battles [Table 3.24] in the campaign, except for the battle of Coulmiers in which the attacking French army defeated the Prussian army. This is the only successful battle the French army fought in the campaign. Note that the French army had three times more forces than the Prussian forces in the battle Coulmiers, which ended in French victory.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	2	1	1	3				3
Number of Battles Attacker Wins (a)	2	0	1	3				3
P(Attacker wins given FR)	**1**	**0**	**1**	**1**				**1**

Table 3.23. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The battle of Sedan [Table 3.24] in 1870 was a milestone in German history. In this battle, Germany decisively defeated the French and dominated Europe. In the battle of Sedan, the Prussian army of 200,000 under the command of Moltke attacked the French army of 120,000, led by Macmahon. The battle itself was the most heavily concentrated combat of the campaign.

Battle Name	**Time**	**Total Strength of the attacker A**	**Total Strength of the defender D**	**Win case {1 if A, -1 if D, 0 o.w.}**	**Force Ratio(FR) A / D**
WEISSENBURG	1870	51000	6000	1	**8.500**
FROESCHWILLER	1870	82000	41000	1	**2.000**
SPICHERN	1870	42000	28000	1	**1.500**
MARS LA TOUR	1870	91000	113000	1	**0.805**
GRAVELOTTE	1870	187000	113000	1	**1.655**
SEDAN	1870	200000	120000	1	**1.667**
COULMIERS	1870	60000	20000	1	**3.000**
ORLEANS	1870	86000	116000	1	**0.741**
LE MANS	1871	72000	88000	1	**0.818**
BELFORT	1871	110000	40000	-1	**2.750**

Table 3.24. FR Value of Each Battle.

O. WORLD WAR I (WWI)—(1914-1918)

1. History of the Campaign

The long-building arms race and hostile alliances among the major powers of Europe finally erupted into war [Ref 3.3] in 1914. Germany and Austria, soon to be called the Central Powers, were joined by the Ottoman Empire and Bulgaria. Serbia, Russia, Belgium, France, and Great Britain came to be known as the Allies. The fighting raged around the world leaving almost 10,000,000 dead and 20,000,000 wounded. A series of treaties, signed in 1919, ended the conflict all over the world.

2. Analysis of Force Ratio

The World War I (WWI) as a campaign is one of the major group of 124 battles [Table 3.17] that may reveal important trends of the FR analysis. WWI and World War II (WWII) dominate the data set comprising of 315 battles out of 552.

In WWI, 74 out of 124 battles [Table 3.25] were fought under the numerical superiority of the attackers, that is, the attacker outnumbered the defender. Interestingly, 44 of these 74 battles [Table 3.25] ended in the attackers' success. Also, 28 more battles were fought under the numerical superiority of the attackers having FR values between 1 and 1.3.

The hypothesis that the smaller the FR values, the less the win probability of the attacker appears roughly with FR values 3, 2.5,2, and 1.5. As the FR value becomes smaller, the P(Attacker wins given FR) decreases accordingly. The campaign was a major group of battles. The campaign might convey general trends referring to the FR.

Although P(Attacker wins) values of higher FR values are greater than those of smaller FR values, say 0.64 of FR=3 and 0.5833 of FR=2, no significant numerical differences exit.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	25	6	17	26	3	4	21	22
Number of Battles Attacker Wins (a)	16	4	8	16	1	2	12	7
P(Attacker wins given FR)	**0.64**	**0.667**	**0.4706**	**0.6154**	**0.333**	**0.5**	**0.571**	**0.318**

Table 3.25. P(Attacker wins given FR) Values Corresponding to Each FR Category.

The campaign is full of battles that include a wide range of unit sizes from 155 in the battle of Medeah Farm to 1,000,000 troops in the battle of Aisne II [Table 3.26]. Apparently, deriving comprehensive trends among widely split unit sizes including general trends in the campaign is difficult. However, the number of battles in the campaign is large enough to exhibit trends in the FR.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
ALSACE-LORRAINE I	1914	457000	345000	-1	**1.325**
ALSACE-LORRAINE II	1914	350000	400000	1	**0.875**
THE ARDENNES	1914	360000	400000	-1	**0.900**
THE SAMBRE	1914	440000	254000	1	**1.732**
MONS	1914	260000	70000	1	**3.714**
LE CATEAU	1914	250000	40000	1	**6.250**
GUISE	1914	260000	200000	0	**1.300**
HEIGHTS OF NANCY	1914	350000	276000	-1	**1.268**
OURCQ I	1914	100000	45000	0	**2.222**
OURCQ II	1914	198000	157000	0	**1.261**
PETIT MORIN	1914	227000	82000	1	**2.768**
TWO MORINS	1914	90000	13000	1	**6.923**
MARSHES OF ST.GOND	1914	101000	141000	-1	**0.716**
VITRY LE FRANCOIS	1914	113000	170000	-1	**0.665**
GAP OF REVIGNY	1914	142000	180000	-1	**0.789**
THE AISNE	1914	343000	290000	-1	**1.183**
EASTERN CHAM	1915	163182	85220	-1	**1.915**
NEUVE CHAPELLE	1915	87000	40000	-1	**2.175**
YPRES II	1915	150000	190000	-1	**0.789**
FESTUBERT	1915	90365	30000	-1	**3.012**
LOOS	1915	298437	75000	-1	**3.979**
STALLUPONEN	1914	50000	40000	-1	**1.250**
GUMBINNEN	1914	120000	150000	0	**0.800**
TANNENBERG	1914	187000	160000	1	**1.169**
MASURIAN LAKES	1914	288600	273000	1	**1.057**
KRASNIK	1914	350000	260000	1	**1.346**
KOMAROV	1914	300000	260000	1	**1.154**
GNILA LIPA	1914	240000	480000	-1	**0.500**
RAVA RUSSKA	1914	900000	936000	-1	**0.962**
LODZ	1914	260000	400000	1	**0.650**
THE JADAR	1914	200000	200000	-1	**1.000**
THE KOLUBRA	1914	200000	300000	1	**0.667**
WINTER BATTLE	1915	650000	300000	1	**2.167**
GOLICE-TARNOW	1915	216000	219000	1	**0.986**

Table 3.26. FR Value of Each Battle.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
FIRST ISONZO	1915	200000	100000	-1	**2.000**
SECOND ISONZO	1915	200000	128500	-1	**1.556**
THIRD ISONZO	1915	356000	157000	-1	**2.268**
FOURTH ISONZO	1915	311000	136000	-1	**2.287**
FIRST DARDANELLES	1915	32000	10000	0	**3.200**
SUVLA BAY	1915	25000	15800	-1	**1.582**
KUT-EL-AMARA	1915	11000	11300	1	**0.973**
CTESIPHON	1915	13756	20400	-1	**0.674**
FIRST SOMME	1916	600000	300000	-1	**2.000**
SOMME-FOURTH ARMY	1916	290000	95000	-1	**3.053**
SOMME-OVILLERS	1916	11300	2800	-1	**4.036**
SOMME-BAZENTIN	1916	45000	15000	0	**3.000**
SOMME-FLERS	1916	190000	90000	1	**2.111**
CAUCASUS WINTER	1916	103000	61000	1	**1.689**
LAKE NAROTCH	1916	350000	180000	-1	**1.944**
1916 BRUSILOV	1916	600000	500000	1	**1.200**
FIFTH ISONZO	1916	300000	160000	-1	**1.875**
ASIAGO	1916	213000	118000	1	**1.805**
TRENTINO COUNTER	1916	200000	172000	1	**1.163**
SIXTH ISONZO	1916	308000	168000	1	**1.833**
ARRAS	1917	276000	120000	0	**2.300**
AISNE II	1917	1000000	480000	-1	**2.083**
MESSINES	1917	180000	100000	1	**1.800**
YPRES III	1917	380000	200000	-1	**1.900**
CAMBRAI I	1917	90000	75000	1	**1.200**
CAMBRAI II	1917	130000	90000	1	**1.444**
TENTH ISONZO	1917	280000	165000	1	**1.697**
ELEVENTH ISONZO	1917	518000	252000	1	**2.056**
CAPORETTO	1917	602000	574000	1	**1.049**
TIGRIS CROSSING	1917	46000	10500	1	**4.381**
GAZA I	1917	25000	26000	-1	**0.962**
GAZA II	1917	25000	20000	-1	**1.250**
GAZA III	1917	72000	34000	1	**2.118**
JUNCTION STATION	1917	85000	15500	1	**5.484**
SECOND SOMME	1918	800000	400000	1	**2.000**
SECOND SOMME	1918	700000	600000	0	**1.167**
LYS	1918	500000	400000	1	**1.250**
YVONNE & ODETTE	1918	3072	650	1	**4.726**
CHEMIN-DES-DAMES	1918	250000	75000	-1	**3.333**
CANTIGNY	1918	8679	725	1	**11.971**
BELLEAU WOOD	1918	9437	6436	0	**1.466**
HILL 142	1918	2913	2458	1	**1.185**
WEST WOOD I	1918	1740	1121	-1	**1.552**
BOURESCHES I	1918	2733	1352	0	**2.021**
HILL 192	1918	3608	3955	-1	**0.912**

Table 3.26. (continued) FR Value of Each Battle.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio(FR) A / D
WEST WOOD II	1918	3343	1798	1	**1.859**
NORTH WOOD I	1918	1747	1952	1	**0.895**
BOURESCHES II	1918	3690	2629	-1	**1.404**
NORTH WOOD II	1918	1697	1428	-1	**1.188**
NORTH WOOD III	1918	1256	1565	-1	**0.803**
NORTH WOOD IV	1918	4453	1546	1	**2.880**
VAUX	1918	12812	10358	1	**1.237**
LA ROCHE WOOD	1918	4515	5182	1	**0.871**
LA ROCHE WOOD	1918	4508	5177	1	**0.871**
NOYON-MONTDIDIER	1918	275000	300000	-1	**0.917**
CHAMPAGNE-MARNE	1918	400000	450000	-1	**0.889**
AISNE-MARNE I	1918	750000	450000	1	**1.667**
MISSY AUX BOIS	1918	5004	3013	1	**1.661**
BREUIL	1918	5039	2663	1	**1.892**
ST. AMAND FARM	1918	1150	400	1	**2.875**
BEAUREPAIRE FARM	1918	4480	565	1	**7.929**
CRAVANCON FERME	1918	10345	2420	1	**4.275**
CHAUDUN	1918	1611	800	1	**2.014**
AISNE-MARNE II	1918	725000	400000	1	**1.813**
BERZY LE SEC	1918	4000	350	1	**11.429**
BUZANCY RIDGE	1918	5300	554	1	**9.567**
PICARDY 1918, I	1918	225000	170000	1	**1.324**
PICARDY 1918, II	1918	300000	200000	1	**1.500**
ST. MIHIEL	1918	400000	100000	1	**4.000**
LAHAYVILLE-BOIS	1918	13208	2090	1	**6.320**
MEUSE-ARGONNE I	1918	300000	190000	1	**1.579**
BLANC MONT I	1918	26000	13000	1	**2.000**
MEDEAH FARM	1918	1921	155	1	**12.394**
ESSEN HOOK	1918	1420	216	1	**6.574**
BLANC MONT RIDGE	1918	1400	458	1	**3.057**
SOMMEPY WOOD	1918	9230	670	1	**13.776**
BLANC MONT II	1918	18000	10000	0	**1.800**
MEUSE-ARGONNE II	1918	500000	300000	1	**1.667**
EXERMONT-MONT	1918	5336	3245	0	**1.644**
MAYACHE RAVINE	1918	5427	1899	0	**2.858**
LA NEUVILLE	1918	5365	1940	1	**2.765**
FERME DES GRANGES	1918	5461	2587	1	**2.111**
HILL 212	1918	5022	3335	1	**1.506**
BOIS DE BOYON	1918	4778	2925	1	**1.634**
HILL 272	1918	2950	2563	1	**1.151**
MEUSE-ARGONNE III	1918	600000	380140	1	**1.578**
REMILLY-AILLICOURT	1918	1210	296	1	**4.088**
HILL 252-PONT	1918	1989	1655	1	**1.202**
THE PIAVE	1918	840000	784000	-1	**1.071**
MEGIDDO	1918	51170	18250	1	**2.804**

Table 3.26. (continued) FR Value of Each Battle.

As one of the major campaigns in the data set, WWI has 102 battles that have FR values greater than one. As mentioned in the very early hypothesis, evidently the P(Attacker wins given FR) value increases as the FR value increases. To test this hypothesis, a model is [Ref 3.6] established:

H_o: $\beta_1 = 0$ (There is no relationship between FR and P(Win))

H_a: $\beta_1 \neq 0$ (There is relationship between FR and P(Win)) (3.5)

If a relationship between the battle outcome and the FR value is proved, the hypothesis that a relationship exists between the battle outcome and the FR value, as claimed in the null hypothesis (3.5), is also proved. The resulting Logistics regression is:

```
Coefficients:
(Intercept)     FR
0.02204063 0.3411387
```

0.34 is our estimated coefficient (β_1) associated with FR. A positive β_1 means that the probability of winning increases as FR increases. The p-value of 0.11 says that we would see data like this 11% of the time if there were no relationship. This suggests a relationship, however we would not reject H_o at the 0.05 significance level. Since the number of battles in WWI is relatively large, the results are more definitive than those of the Napoleonic Wars and the American Civil War.

```
1-pchisq(103.7902,88)
[1] 0.1199488
```
(3.6)

P. WORLD WAR II (WWII)—(1939-1945)

1. History of the Campaign

When Hitler invaded Poland [Ref 3.3] on September 1, 1939, Great Britain and France, renouncing their previous policy of appeasement, declared war on Germany two days later. Mainly Great Britain, France, Russia, and later the U.S.A. established Allied Forces against Axis Forces of Germany, Italy, and Japan.

Battles were fought all around the world from the Pacific to the Atlantic Ocean and from Europe to Africa, claiming the lives of approximately 14 million people. WWII ended with the surrender of Japan on August 14, 1945.

2. Analysis of Force Ratio

WWII establishes the biggest group of battles in the data set comprising 191 of 552 battles [Table 3.27]. Some analysts start the era of modern warfare from WWII so that any emerging trend of FR can be translated into modern warfare. Roughly speaking, the smaller the FR, the lower the probability for the attacker to win. Thus, the proposition holds for the FR values of 3, 2.5, 2, and 1.5 respectively.

Also, noteworthy is that 171 out of 191 battles [Table 3.27] were fought under the numerical superiority of the attackers. That is, the attackers were more numerous than the defender. 104 of these 191 battles [Table 3.27] ended in the attackers' success.

WWII is one of two campaigns that include P(Attacker wins given FR) for all of the FR portions. In this respect, WWI and WWII establish the domain set of the general trend governing the FR. A comparison of two campaigns reveals some similarities among the FR values. Tracking the FR values from FR = 3 or more cases to the right of the table, one sees a roughly decreasing trend in the P(Attacker wins given FR) values from FR=3

or more to FR=2-1.5 despite the local increases in-between. However, a sharp decrease in the P(Attacker wins given FR) takes place at the point of FR=1.5 showing a marginally increasing characteristic to the right of the point. This interesting feature of the P(Attacker wins given FR) values is examined in the conclusions.

WWI	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	25	6	17	26	3	4	21	22
Number of Battles Attacker Wins (a)	16	4	8	16	1	2	12	7
P(Attacker wins given FR)	**0.64**	**0.667**	**0.4706**	**0.6154**	**0.333**	**0.5**	**0.571**	**0.318**

WWII	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	66	25	30	27	4	10	13	16
Number of Battles Attacker Wins (a)	47	17	23	17	1	4	5	5
P(Attacker wins given FR)	**0.712**	**0.68**	**0.7667**	**0.63**	**0.25**	**0.4**	**0.38**	**0.313**

Table 3.27. P(Attacker wins given FR) Values Corresponding to Each FR Category.

It is an early indication of the general trend in the P(Attacker wins given FR) corresponding to the FR values that the P(Attacker wins given FR) value decreases as the FR value decreases. Also, seemingly it is quite logical to separate the P(Attacker wins given FR) corresponding to the FR values into two groups [Table 3.27]: before and after the FR=1.5. Because a noticeable decrease in the P(Attacker wins given FR) values occur at the point of the FR=1.5 increasing marginally to the right of that point.

The unit sizes in the campaign change widely from 1,250,000 attacking Russian troops in the battle of Yassy Kishinev [Table 3.28] in 1944 to 188 defending American troops in the battle of the Chouigui Pass in 1942. This disparity in unit sizes in the campaign makes the general trend governing the P(Attacker wins given FR) values corresponding to the FR's more comprehensive than before.

Actually, WWII can be divided into subgroups regarding the regions where the battles occured. That approach may reveal the regional trends of the P(Attacker wins

given FR) values to an extent. Furthermore, the probable characteristics of the fighting forces can be derived related to the region and the FR. However, this may distort the overall picture of the general trends that correspond to the P values of the FR's. This approach should be applied in more specific analysis of the topic.

Being the biggest group of battles in the data set, the hypothesis testing is applied to WWII to evaluate whether a relationship between battle outcomes and the FR values exists. Since the P(Attacker wins given FR) value increases as the FR value increases, the hypothesis testing for the model [Ref 3.6] is:

H_o: $\beta_1 = 0$ (There is no relationship between FR and P(Win).)

H_a: $\beta_1 \neq 0$ (There is relationship between FR and P(Win).) (3.7)

The Logistics regression [Ref 3.2] analysis is applied to the model (3.7) so that the p-value concerning the model can be determined and compared to the α=0.05. If the p-value of the model is greater than the alpha level of 0.05, the null model that indicates no relationship between battle outcomes and the FR values is accepted as the model governing the campaign.

```
(Intercept)    FR
0.2661204 0.2339534
```

The p-value of 0.07 says that we would see data like this 7% of the time if there were no relationship. This suggests a relationship, however we would not reject H_o at the 0.05 significance level. Since the number of battles in WWII is relatively large, the results are more definitive than those of WWI.

```
1-pchisq(181.9276,156)
[1] 0.07623339.
```
(3.8)

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
SEDAN-MEUSE RIVER	1940	48000	60000	1	**0.800**
CAMBRAI	1940	17,000	12,143	1	**1.400**
ARRAS	1940	11,821	18,000	-1	**0.657**
BOOS	1940	189	189	0	**1.000**
JITRA	1941	7000	12000	1	**0.583**
ROVNO	1941	132000	150000	1	**0.880**
DEFENSE OF MOSCOW	1941	1100000	1372000	-1	**0.802**
MOSCOW COUNTER	1941	1060300	880000	1	**1.205**
ALAM HALFA	1942	124000	120000	-1	**1.033**
EL ALAMEIN II	1942	220476	105223	1	**2.095**
OPERATION LIGHTFOOT	1942	220476	105223	1	**2.095**
ALAMEIN BRIDGEHEAD	1942	214336	101528	1	**2.111**
OPERATION SUPER	1942	211000	97000	1	**2.175**
CHOUIGUI PASS	1942	465	188	-1	**2.473**
POGORELOYE GOROD	1942	54180	12035	1	**4.502**
EL GUETTAR	1943	10300	22019	-1	**0.468**
SEDJANNE-BIZERTE	1943	24098	5000	1	**4.820**
AMPHITHEATER	1943	12917	4250	1	**3.039**
PORT OF SALERNO	1943	12917	4250	1	**3.039**
SELE-CALORE	1943	12447	8390	-1	**1.484**
BATTIPAGLIA I	1943	14730	11230	-1	**1.312**
VIETRI	1943	15000	12917	-1	**1.161**
TOBACCO FACTORY	1943	14733	12691	-1	**1.161**
BATTIPAGLIA II	1943	14730	6995	1	**2.106**
EBOLI	1943	15576	6702	1	**2.324**
VIETRI II	1943	13300	18912	-1	**0.703**
GRAZZANISE	1943	14557	8068	1	**1.804**
CAIAZZO	1943	18210	6435	1	**2.830**
CAPUA	1943	16857	8000	-1	**2.107**
CASTEL VOLTURNO	1943	17765	8158	1	**2.178**
MONTE ACERO	1943	21265	6435	1	**3.305**
TRIFLISCO	1943	18476	7250	1	**2.548**
DRAGONI	1943	17034	5152	-1	**3.306**
CANAL I	1943	14600	8138	1	**1.794**
MONTE GRANDE	1943	16400	7239	1	**2.266**
CANAL II	1943	17500	8128	1	**2.153**
FRANCOLISE	1943	14000	8088	-1	**1.731**
SANTA MARIA	1943	16870	6321	1	**2.669**
MONTE CAMINO I	1943	19513	6750	-1	**2.891**
MONTE LUNGO	1943	16600	6566	-1	**2.528**
POZZILLI	1943	17404	6566	-1	**2.651**
MONTE CAMINO II	1943	7942	5200	1	**1.527**
MONTE ROTONDO	1943	16350	7942	0	**2.059**
CALABRITTO	1943	17765	7588	-1	**2.341**
MONTE CAMINO III	1943	20744	3288	1	**6.309**
MONTE MAGGIORE	1943	5551	3288	1	**1.688**
LENINGRAD	1943	120000	30000	1	**4.000**
OBOYAN-KURSK	1943	62000	45000	1	**1.378**
OPERATION CITADEL	1943	140000	75000	1	**1.867**
OBOYAN-KURSK I	1943	60000	149000	1	**0.403**
OBOYAN-KURSK II	1943	56000	129000	-1	**0.434**
PROKHOROVKA	1943	78000	82300	1	**0.948**
KURSK COUNTER	1943	980600	280000	1	**3.502**

Table 3.28. FR Value of Each Battle.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case (1 if A, -1 if D, 0 o.w.)	Force Ratio (FR) A / D
BELGOROD	1943	70000	15000	1	4.667
MELITOPOL	1943	524724	210000	1	2.499
TARAWA-BETIO	1943	9000	4836	1	1.861
SIDI BOU ZID I	1943	6,400	5,333	1	1.200
SIDI BOU ZID II	1943	2,738	8,380	-1	0.327
KASSERINE PASS	1943	7,000	5,303	1	1.320
APRILIA I	1944	19350	6750	1	2.867
THE FACTORY	1944	15317	17976	-1	0.852
CAMPOLEONE	1944	17766	15098	1	1.177
CAMPOLEONE	1944	26029	9834	1	2.647
CARROCETO	1944	26490	4515	-1	5.867
MOLETTA RIVER	1944	7418	5000	0	1.484
APRILIA II	1944	27518	17730	1	1.552
FACTORY COUNTER	1944	13400	7077	-1	1.893
BOWLING ALLEY	1944	41974	20496	-1	2.048
MOLETTA RIVER II	1944	21478	9761	1	2.200
FIOCCIA	1944	15637	19613	-1	0.797
SANTA MARIA	1944	18702	9250	1	2.022
SAN MARTINO	1944	17970	8141	1	2.207
CASTELLONORATO	1944	16458	7500	1	2.194
SPIGNO	1944	18308	8215	1	2.229
FORMIA	1944	23190	7627	1	3.041
MONTE GRANDE	1944	13095	4563	1	2.870
ITRI-FONDI	1944	17912	6653	1	2.692
TERRACINA	1944	18030	6653	1	2.710
MOLETTA OFFENSIVE	1944	17345	12569	0	1.380
ANZIO-ALBANO	1944	17313	11343	0	1.526
ANZIO BREAKOUT	1944	22374	12815	1	1.746
CISTERNA	1944	19971	11928	1	1.674
SEZZE	1944	17925	6957	1	2.577
VELLETRI	1944	20683	12327	-1	1.678
CAMPOLEONE	1944	19047	10593	0	1.798
VILLA CROCETTA	1944	18000	13715	-1	1.312
ARDEA	1944	15557	7659	1	2.031
FOSSO DI CAMP	1944	29711	15801	-1	1.880
LANUVIO	1944	17300	6108	-1	2.832
LARIANO	1944	22641	13012	1	1.740
VIA ANZIATE	1944	23604	19255	0	1.226
VALMONTONE	1944	26607	10111	1	2.631
TARTO-TIBER	1944	38011	10855	1	3.502
IL GIOGIO PASS	1944	15721	3700	1	4.249
ST. LO	1944	18228	7500	1	2.430
OPERATION GOOD	1944	76213	57500	-1	1.325
OPERATION COBRA	1944	126000	30700	1	4.104
MORTAIN	1944	25497	27673	-1	0.921
CHARTRES	1944	15646	8325	0	1.879
MELUN	1944	17232	6000	1	2.872
SEINE RIVER	1944	40619	15000	1	2.708
MOSELLE-METZ	1944	59631	41500	0	1.437
METZ	1944	60794	39580	-1	1.536
ARRACOURT	1944	7500	4800	-1	1.563
WESTWALL	1944	32283	19632	1	1.644
SCHMIDT	1944	20493	20250	-1	1.012
SEILLE-NIED	1944	99583	23588	1	4.222
FORET DE CHATEAU	1944	43587	11185	1	3.897
MORHANGE	1944	25881	7555	1	3.426
MORHANGE-FAUL	1944	92393	28382	1	3.255
BOURGALTROFF	1944	10348	6519	1	1.587
SARRE-ST. AVOLD	1944	88941	32396	1	2.745
BAERENDORF I	1944	7935	5366	1	1.479
BAERENDORF II	1944	15871	6999	1	2.268
BURBACH-DURSTEL	1944	16232	6713	1	2.418
DURSTEL-FAERBER	1944	90078	30712	0	2.933
SARRE-UNION	1944	19773	6044	1	3.272
SARRE-SINGLING	1944	89977	31501	1	2.856
SINGLING-BINING	1944	15224	5044	0	3.018
SAUER RIVER	1944	10000	8634	1	1.158
ST. VITH	1944	87000	19996	0	4.351
BASTOGNE	1944	36678	4849	-1	7.564
KORSUN-SCHEVCHEN	1944	254950	84500	1	3.017
NIKOPOL BRIDGEHEAD	1944	25100	8230	1	3.050
SEVASTOPOL	1944	397607	72000	1	5.522
BEREZINA RIVER	1944	16100	8500	1	1.894
LVOV-SANDOMIERZ	1944	1200000	900000	1	1.333
BRODY (PHASE I)	1944	39000	3300	1	11.818
BRODY (PHASE II)	1944	38500	12900	1	2.984
VISTULA RIVER, I	1944	12700	5100	1	2.490
VISTULA RIVER, II	1944	17550	6400	-1	2.742
YASSY-KISHINEV	1944	1250000	800000	1	1.563

Table 3.28. (continued) FR Value of Each Battle.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
BOWLING ALLEY II	1944	15,736	5,050	1	**3.116**
BOWLING ALLEY III	1944	10,000	4,625	-1	**2.162**
MORTAIN I	1944	8,150	3,700	-1	**2.203**
MORTAIN II	1944	8,500	4,600	-1	**1.848**
SCHMIDT I	1944	6,200	5,025	-1	**1.234**
SCHMIDT II	1944	4,350	3,450	-1	**1.261**
SCHMIDT III	1944	4,950	3,700	-1	**1.338**
WAHLERSCHEID	1944	8,300	1,400	1	**5.929**
KRINKELT-ROCHERATH I	1944	3,300	1,357	1	**2.432**
KRINKELT-ROCHERATH II	1944	9,100	6,600	-1	**1.379**
SCHNEE EIFEL CENTER	1944	4,100	3,900	1	**1.051**
SCHNEE EIFEL SOUTH	1944	11,000	4,300	1	**2.558**
SCHNEE EIFEL NORTH I	1944	14,300	2,050	1	**6.976**
SCHNEE EIFEL NORTH II	1944	12,800	4,150	1	**3.084**
OUR RIVER CENTER	1944	43,800	5,340	1	**8.202**
TARGUL FRUMOS	1944	35,170	13,725	-1	**2.562**
TARTO-TIBER	1944	38,011	10,855	1	**3.502**
VISTULA-ODER	1945	2200000	560000	1	**3.929**
EAST PRUSSIA	1945	1220000	780000	1	**1.564**
CIECHANOW (PHASE I)	1945	10800	3100	0	**3.484**
CIECHANOW (PHASE II)	1945	12115	3900	1	**3.106**
SEELOW HEIGHTS	1945	13600	3710	1	**3.666**
MUTANKIANG	1945	147000	75000	1	**1.960**
IWO JIMA	1945	33915	18300	1	**1.853**
IWO JIMA - SURIBACHI	1945	3200	1600	1	**2.000**
IWO JIMA - FINAL PHASE	1945	32000	2685	1	**11.918**
BEACHHEAD	1945	22888	1400	1	**16.349**
OUTPOSTS	1945	18398	2900	1	**6.344**
TOMB HILL-OUKI	1945	18111	4731	1	**3.828**
SKYLINE RIDGE-ROCKY	1945	16291	2600	1	**6.266**
KOCHI RIDGE-ONAGA I	1945	14594	5000	-1	**2.919**
KOCHI RIDGE-ONAGA II	1945	15986	4500	-1	**3.552**
KOCHI RIDGE-ONAGA III	1945	15764	4050	-1	**3.892**
JAPANESE COUNTER	1945	6850	15350	-1	**0.446**
KOCHI RIDGE IV	1945	15109	5140	1	**2.939**
SHURI (PHASE I)	1945	16043	3338	1	**4.806**
JAPANESE COUNTER	1945	4000	15777	-1	**0.254**
SHURI (PHASE II)	1945	15840	3000	-1	**5.280**
SHURI (PHASE III)	1945	15205	2600	1	**5.848**
HILL 95-I	1945	16091	3500	0	**4.597**
HILL 95-II	1945	16002	2500	1	**6.401**
YAEJU-DAKE	1945	5237	2500	1	**2.095**
HILLS 153 AND 115	1945	15808	2000	1	**7.904**
ADVANCE	1945	19082	2000	1	**9.541**
ADVANCE TO SHURI	1945	18388	2900	1	**6.341**
KAKAZU AND TOMBSTONE	1945	21247	3000	-1	**7.082**
NISHIBARU RIDGE	1945	17163	3000	1	**5.721**
MAEDA ESCARPMENT	1945	18095	3900	1	**4.640**
ATTACK ON SHURI	1945	19714	5284	0	**3.731**
ATTACK ON SHURI FLANK II	1945	20973	4757	0	**4.409**
ATTACK ON SHURI FLANK III	1945	19658	4227	1	**4.651**
ADVANCE TO YUZA-DAKE	1945	18777	4000	1	**4.694**
ATTACK ON YUZA-DAKE	1945	18660	4250	0	**4.391**
CAPTURE OF YUZA-DAKE	1945	19047	3250	1	**5.861**

Table 3.28. (continued) FR Value of Each Battle.

Q. KOREAN WAR—(1950-1951)

1. History of the Campaign

Secretly armed with Russian and Chinese equipment, the 127,000-man army of North Korea suddenly advanced [Ref 3.3] across the 38th parallel on June 25, 1950. The 98000-troop army of South Korea, advised by a 500-man American military group, was caught unprepared.

The following day, the United Nations (U.N.) voted to provide military aid. 15 nations battled North Korea under the U.N. banner. Finally, an armistice on July 27, 1953, ended open hostilities with a heavily manned border along the 38th parallel still cutting the peninsula in half.

2. Analysis of Force Ratio

The Korean War stands as a counter example to the hypothesis claiming that the smaller the FR values are, the less the winning probability of the attacker is. Ignoring the 3-to-1 because of the NA, P (Attacker wins given FR) values become greater as the FR values become smaller [Table 3.29].

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)		1	2	2			1	5
Number of Battles Attacker Wins (a)		0	2	2			1	4
P(Attacker wins given FR)		**0**	**1**	**1**			**1**	**0.8**

Table 3.29. P(Attacker wins given FR) Values Corresponding to Each FR Category.

Since the number of battles in this campaign is smaller than campaigns such as WWI, the effect of the Korean War over the final results will be relatively small. The attackers won five out of six battles [Table 3.30] in which the attackers had numerical superiority. In the battle of the Iron Triangle in 1951, the North Korean forces attacked the U.S. forces, which defeated the Koreans.

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
PUSAN PERIMETER	1950	11000	15200	-1	**0.724**
PUSAN BREAKOUT	1950	16600	10300	1	**1.612**
NAM RIVER	1950	16400	9000	1	**1.822**
KUNSON	1950	16200	7100	1	**2.282**
BUTTE LINE	1951	29000	30200	1	**0.960**
HAN RIVER	1951	25500	27000	1	**0.944**
CHAN RIVER	1951	26000	12500	1	**2.080**
PIERCE LINE	1951	27900	35100	1	**0.795**
KANSAS LINE	1951	30700	26900	1	**1.141**
IRON TRIANGLE	1951	37000	13800	-1	**2.681**
BAYONETTE LINE	1951	13700	35500	1	**0.386**

Table 3.30. FR Value of Each Battle.

R. ARAB-ISRAELI WAR—1973

1. History of the Campaign

This is the third war [Ref 3.3] between Arab nations and Israel since the independence of Israel. Also, it is the third of three successful Israeli campaigns. It is known as the Yom Kippur War by Israel and the Ramadan War by Arab nations since the war was triggered in one of the Israeli festivals. Israel dominated all fronts with their air superiority.

2. Analysis of Force Ratio

Interestingly, the attackers won just two of eleven battles in which attackers had numerical superiority over the defenders [Table 3.31]. This campaign can be accepted as

an outlier or biased example because Israel's aerial domination gave it a tremendous advantage.

Also, the qualitative features of the Israeli army, such as discipline, and battle strategy contributed to their victory. In the battle of Tel Farris [Table 3.32] in 1973, for example, the Israeli army attacked numerically superior Syrian forces in their defensive positions and gained a clear victory. The same Israeli army successfully defeated the numerically superior Jordanian army in the battle of Naba in 1973. Such examples prove Israeli domination in the campaign.

	FR = 3 or more	FR = 3 - 2.5	FR = 2.5 - 2	FR = 2 - 1.5	FR = 1.5 - 1.4	FR = 1.4 - 1.3	FR = 1.3-1	FR = 1 or less
Number of Battles (n)	5		3	3			3	15
Number of Battles Attacker Wins (a)	1		1	0			1	11
P(Attacker wins given FR)	**0.2**		**0.333**	**0**			**0.333**	**0.733**

Table 3.31. P(Attacker wins given FR) Values Corresponding to Each FR Category.

Meanwhile, the force sizes of this campaign reflect another important feature of modern warfare: they are relatively smaller than before. In modern warfare most battles are more likely between relatively small forces than fewer battles between large battle formations. An example of modern warfare was Arab–Israeli War in 1973, which consisted of numerous small battles. The unit sizes varied between 81,160 in the battle of the Egyptian offensive and 4,750 in two battles near the Mount Hermon [Table 3.32].

Battle Name	Time	Total Strength of the attacker A	Total Strength of the defender D	Win case {1 if A, -1 if D, 0 o.w.}	Force Ratio (FR) A / D
KANTARA-FIRDAN	1973	25850	67440	-1	0.383
EGYPTIAN OFFENSIVE	1973	81160	43400	-1	1.870
EGYPTIAN OFFENSIVE	1973	57960	28600	-1	2.027
DEVERSOIR	1973	22790	30970	1	0.736
DEVERSOIR	1973	28900	36840	1	0.784
DEVERSOIR WEST	1973	19600	18180	1	1.078
ISMAILIA	1973	17000	23860	-1	0.712
JEBEL GENEIFA	1973	16200	35633	1	0.455
SHALLUFA I	1973	16200	25600	1	0.633
SHALLUFA II	1973	11700	22570	1	0.518
SUEZ	1973	14681	22570	-1	0.650
ADABIYA	1973	10900	14620	1	0.746
KUNEITRA	1973	17750	3630	0	4.890
AHMADIYEH	1973	22750	5745	-1	3.960
RAFID	1973	19525	4958	1	3.938
YEHUDA-EL AL	1973	21984	6300	-1	3.490
NAFEKH	1973	12500	6946	-1	1.800
TEL FARRIS	1973	17833	23750	1	0.751
HUSHNIYAH	1973	12733	14683	1	0.867
MOUNT HERMONIT	1973	31650	5395	-1	5.867
MOUNT HERMON I	1973	2692	1583	-1	1.701
TEL SHAMS	1973	16100	19400	1	0.830
TEL SHAAR	1973	14700	21500	1	0.684
TEL EL HARA	1973	12500	14300	-1	0.874
KFAR SHAMS	1973	11000	12000	1	0.917
NABA	1973	11500	11000	-1	1.045
ARAB COUNTER	1973	35750	16100	-1	2.220
MOUNT HERMON II	1973	5,700	4,750	-1	1.200
MOUNT HERMON III	1973	11,400	4,750	1	2.400

Table 3.32. FR Value of Each Battle.

S. CONCLUSIONS:

In normal battlefield conditions, the attackers seek numerical superiority so that they can concentrate their forces at critical locations in the battlefield to assure the success of the attack. A battle usually does not take place unless each side believes it has some chance for success. Otherwise, the attacker would avoid taking the initiative while the defender would attempt to reinforce the battlefield to improve their defense.

One important quantitative feature of the battlefield that might reveal the outcome of the battle is the FR. A historical truism is that the attacker must have a definite numerical superiority over the defender to guarantee the success of the offensive. The most famous of the FR values is the 3-to-1 thumb rule indicating that the attacker should be three times as large as the defender for a successful offensive. The validity of this rule, however, has been questioned in modern combat modeling.

Even though it is more probabilistic than other battle outcome predictors [Ref 3.1], the FR is a valid estimator of the battle outcome. Many analysts, however, question the validity of this concept by giving counter historic examples. In 1973, for example, an Israeli force with 788 tanks attacked an Egyptian force with 808 tanks and then crossed the Suez Canal [Ref 3.1]. The FR was almost 1-to-1, but the attacker succeeded. In 1879, Zulu force of 20,000 attacked British force of 5,317 in Ulundi during the Colonial Wars. The British forces repelled the superior Zulu force. The FR was 3.8-to-1.

Many other battles can be listed as examples; however, they are merely examples of exceptions. These battles have their own stories to tell and some special features in the way they were conducted. In 1973, the early destruction of Arab air forces dominated the whole campaign giving Israel an upper hand in defeating its enemies regardless of the

FR. In 1879, on the other hand, British weaponry superiority enabled its forces to defeat ill-armed Zulu forces.

Actually, these two and many other battles that are cited as counter examples against the validity of the FR concept are biased in one way or another. Thus, proposing biased cases as counter examples is not valid. On the other hand, extracting exceptions from the set of battles, the FR concept proves to be quite valid.

The pattern of the FR values draw quite an interesting graph for the overall analysis of the campaigns in the data set. The FR values are basically divided into seven groups [Table 3.33], namely FR = 3 or more, FR = 3-2.5, FR = 2.5-2, FR = 2-1.5, FR = 1.5-1.4, FR = 1.4-1.3, FR = 1.3 or less. The corresponding P(Attacker wins given FR) values are plotted against each group. It is observed that the P(Attacker wins given FR) values corresponding to the FR's have two distinct patterns divided by FR = 1.5.

The P(Attacker wins given FR) values follow a decreasing order from FR = 3 or more cases to FR = 2-1.5 case. At FR = 1.5-1.4, the P(Attacker wins given FR) value decreases sharply to 0.4615 [Table 3.33]. After FR = 1.5-1.4 to FR = 1.3 or less, the corresponding P values draw a roughly increasing pattern remaining below the smallest value of the first group, namely from FR = 3 or more to FR = 2-1.5.

The consistency of the P(Attacker wins given FR) in the first group of the FR values [Table 3.33] is worth mentioning. It takes P(Attacker wins given FR) = 0.6792 at FR = 3 or more with a gradually decreasing pattern. The P(Attacker wins given FR) hits 0.6579 at FR = 3-2.5, 0.6190 at FR = 2.5-2, 0.6136 at FR = 2-1.5. The P(Attacker wins given FR) values decrease consistently in this group of FR's as the FR decreases from 3 to 1.5. In the second group of P(Attacker wins given FR) values corresponding FR = 1.5-

1.4, FR = 1.4-1.3, FR = 1.3 or less, however, the pattern is quite different with respect to changes in P values. The P(Attacker wins given FR) values take 0.4615 at FR = 1.5-1.4, 0.5238 at FR = 1.4-1.3, and 0.4857 at FR = 1.3 or less. The sudden decrease in the P(Attacker wins given FR) value at FR = 1.5-1.4 is important since it may reveal some hidden features of the battlefield.

			FR = 3 or more		FR = 3 - 2.5		FR = 2.5 - 2		FR = 2 - 1.5		FR = 1.5 - 1.4		FR = 1.4 - 1.3		FR = 1.3-1		FR = 1 or less	
Year	Name of Campaign	Total Number of battles N	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a	Number of battles n	Number of battles attacker wins a
1620	30 Years' War	18							1	1					4	4	13	12
1642	English Civil War	6					2	0	1	1							3	2
1689	King Williams' War	8							2	2			1	1	2	1	3	2
1741	Austrian-Success	7											1	1	1	0	5	4
1756	7 Years War	18	1	0	2	2			2	1					1	1	12	7
1775	American-Revol.	14	1	1					1	1					3	0	9	5
1792	War-of-1st-Coal.	14	3	2					2	2	1	1	2	1	2	2	4	1
1800	War-of-2nd-Coal.	7							2	1	1	1					4	1
1805	Napoleonic War	29			2	2	2	1	4	4	1	0	2	1	8	4	10	2
1846	US-Mexico	8			1	0									1	1	6	6
1861	American-Civil	49	3	3			6	3	12	3	3	2	1	1	10	2	14	2
1870	Famco-Russian	10	2	2	1		1	1	3	3							3	3
1914	WWI	124	25	16	6	4	17	8	26	16	3	1	4	2	21	12	22	7
1944	WWII	191	66	47	25	17	30	23	27	17	4	1	10	4	13	5	16	5
1950	Korean war	11			1	0	2	2	2	2					1	1	5	4
1973	Arab-Israel 1973	29	5	1			3	1	3	0					3	1	15	11
	Totals	543	106	72	38	25	63	39	88	54	13	6	21	11	70	34	144	74
Average P(Attacker wins given FR)				0.6792		0.6579		0.6190		0.6136		0.4615		0.5238		0.4857		0.5139

Table 3.33. Average P(Attacker wins given FR) Values of Each Campaign Category.

If the P(Defender wins given FR) value of each FR is examined as the complement of P(Attacker wins given FR) value, the P values of the defender remain below 50% for the first group of the FR's [Figure 3.1] while they are mostly greater than 50% for the second group. This picture shows that the attacker is more likely to succeed in the battlefield if the FR value is greater than 1.5. The P values corresponding to the second group of the FR's [Figure 3.1], however, deserve more analyses to derive general trends.

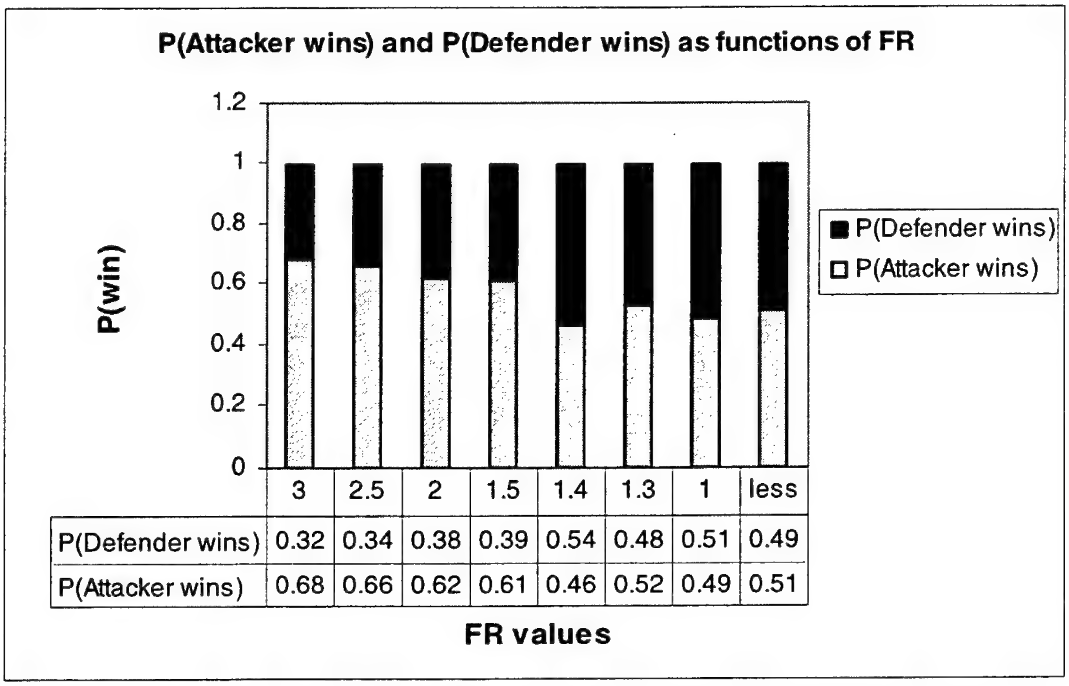

Figure 3.1. P(Attacker wins) and P(Defender wins) Values of Each FR Category.

As the FR becomes less than 1.5, say 1.4, the P(Attacker wins given FR) value decreases instantly to a level of 0.46 while the P(Defender wins given FR) value increases to a level of 0.54. This situation is the first time in the scale that the defender has a greater winning probability than the attacker. When the FR value decreases to FR = 1.3, however, an increase in the P(Attacker wins given FR) takes place: 0.52. At the end of the scale, the P(Attacker wins given FR) value again decreases slightly to the level of 0.49 for the FR = 1.3-1. Of course, those small fluctuations are not statistically significant.

From the tactical perspective in the combat, the beginning and the end values of the FR for the second part of the scale [Figure 3.2], say FR = 1.4-1, have special meaning that may help to clarify the changes in the P(Attacker wins given FR). The defending forces seek chances of counter offensives [Ref 3.4] to drive the attacker away from its

objective in the battlefield. However, the commander of the defending forces should consider the FR value to decide whether he can launch a counter offensive against the attacker. The obvious choice for the defending force commander is to take an offensive action against the attacker if the FR is almost one, indicating that the attacker and the defender have almost the same number of troops. On the other hand, the defending force must hold its positions against the attacking force that is 1.5 or more times larger than its own size.

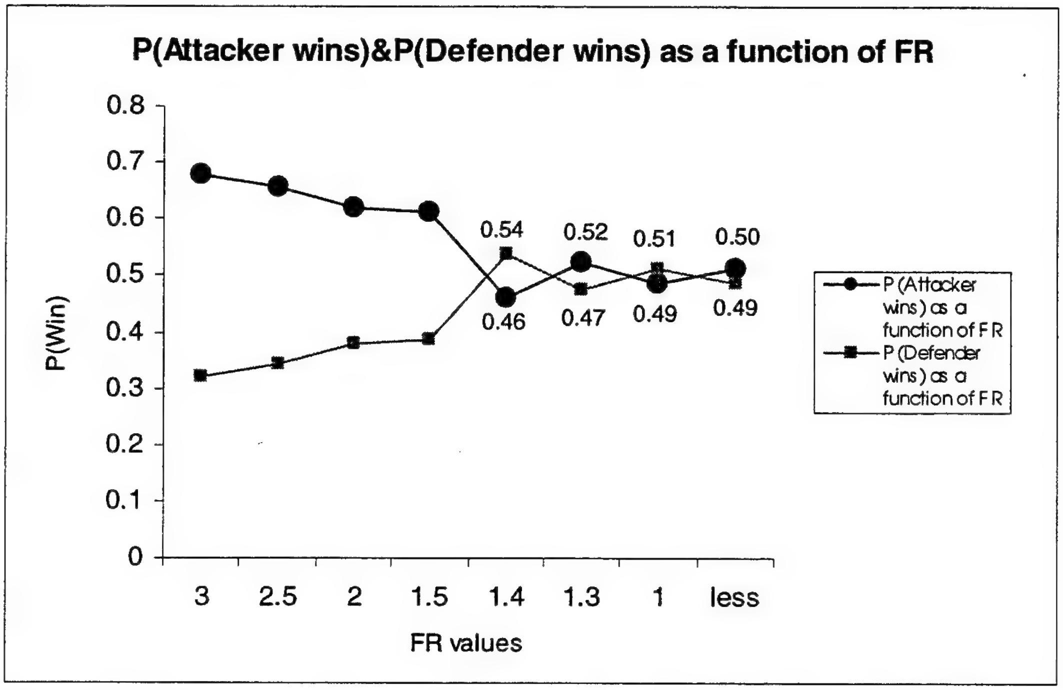

Figure 3.2. P(Attacker wins) and P(Defender wins) Values for Each FR Category.

In the actual battlefield environment, it is very difficult for the defending force commander to assess the size of the attacking force. He can roughly estimate the number of the attacking force with respect to some clues coming from the battlefield. However, computing the size of the attacking force is relatively easy as the FR value increases beyond 1.5. The same rule is applicable for FR values less than or equal to one. In these

clear-cut situations, the defending force commander can easily decide on which action should be taken. However, taking any action is not easy if the force size of the attacking force is between 1 and 1.5 times larger than the defending force. The defending force commander may face a dilemma in selecting any action since the exact size of the attacking force is unknown. Thus, the P(Attacker wins given FR) value increases slightly between FR = 1 and FR = 1.5.

Despite some slight differences among P values corresponding to specific FR values of the data set, the general trend remains applicable for the overall analysis of the campaigns, emphasizing that the P(Attacker wins given FR) value increases as the FR value increases. The difference between consecutive average win probability values are not large enough to eliminate hypothesis testing, but the figures clearly show the trend of win probabilities: the greater the FR is, the higher the win probability of the attacker's given FR is. The very last hypothesis testing is conducted for the overall results of the campaigns.

The Logistics regression [Ref 3.2] analysis is applied to the cumulative results of the campaigns to test the hypothesis that the attacker's winning probability increases as the FR increases. In this phase, the resulting data are analyzed in two cases: FR values greater than 1.5 and FR values smaller than 1.5 since two different characteristics take place in the resulting P values corresponding to FR's. In the first case, a Logistic regression is applied to P values corresponding FR's that are greater than 1.5. The model (3.9) for this part is as follows:

H_o: $\beta_1 = 0$ (There is no relationship between FR and P(Win))

H_a: $\beta_1 \neq 0$ (There is relationship between FR and P(Win)) (3.9)

All battles with FR values that are greater than 1.5 are listed in the data frame to get a comprehensive result for the model. The result of the Logistics regression governs only the first part of the data set that has FR values greater than 1.5.

```
Coefficients:
(Intercept)    FR
 0.2406702 0.2546265
1-pchisq(256.4436,227)
[1] 0.08735407
```

(3.10)

When the p-value (3.10) of the first part is evaluated and compared to the alpha level of 0.05, it is understood that the p-value, 0.087, is greater than the alpha level of 0.05. This result shows a relationship between the battle outcomes and the FR values as stated in the null hypothesis. When the same analysis is applied to the second part of the data, namely FR<1.5, the following result occurs:

```
Coefficients:
 (Intercept)    FR
 -1.302165 1.044493
> 1-pchisq(98.054,69)
[1] 0.01230675
```

(3.11)

In the second part, however, the p-value (3.11) of the model, 0.012, is smaller than the alpha level of 0.05. This result states that the null hypothesis is rejected for the alpha level of 0.05 and there is not a relationship between battle outcomes and the FR values in the second part of the data set. Actually, this result is what is expected for the second part, namely FR=1.5-1. Since there are fluctuations in P(Attacker wins given FR) values corresponding to FR's that are less than 1.5, the model is not expected to fit the second part of the data set. That is why the second part corresponding to FR=1.5-1 is

explained above as a different entity and analyzed with respect to tactical decisions on the battlefield. Although the second part of the data set does not fit the model mentioned above, the fact that the P(Attacker wins given FR) value increases as the FR increases remains valid for the overall analysis of results.

Also, Kendall's Tau (τ) and Spearman's Rho(ρ) [Ref 3.5] can be calculated to figure out if higher FR values tend to be associated with higher P(Attacker wins given FR) values. The model (3.12) for these analyses is:

Ho: The FR and P values are independent.

Ha: There is a tendency for the larger FR's and P's to be paired together. (3.12)

The Kendall's Tau (τ) computation is:

normal-z = 2.5532, p-value = 0.0107 (3.13)

alternative hypothesis: true tau is not equal to 0

sample estimates:

tau

0.8095238

The Spearman's Rho(ρ) computation is:

normal-z = 2.1433, p-value = 0.0321 (3.14)

alternative hypothesis: true rho is not equal to 0

sample estimates:

rho

0.8928571

As it is obvious from both calculations, the null hypothesis that P values and FR's are independent is rejected in both cases since the p-values for the Kendall's Tau (3.13) and Spearman's Rho (3.14) are 0.0107 and 0.0321 respectively, which are much smaller than the alpha level of 0.05. Thus, the alternative hypothesis that there is a tendency for

the larger FR's and P(Attacker wins given FR) values to be paired together gains statistical support.

This section has shown that the probability of winning is higher when FR is higher. We only looked at FR, many other factors, such as the abilities of the combatants, also contribute to the probability of winning. However, these factors can be subjective and may be unknown prior to conflict.

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IV. DISPERSION

A. INTRODUCTION

In this chapter, the concept of "dispersion" (see Chapter IV Part B) is analyzed for the campaign-wise grouping of the CDB90FT data set. The analysis seeks general trends concerning campaigns. Since the specific data of "depth" in the combat is available for only six campaigns in the data set of CDB90FT, the analysis covers only those campaigns, which are the Napoleonic Wars, the American Civil War, the Franco-Russian War, World War I (WWI), World War II (WWII), and the Arab-Israel War of 1973.

While examining the concept of "dispersion," the historical trends are identified and the probable specifications of individual campaigns are given. The focus of the analysis is to look for relationships between the lethality of weapons, the casualty rate and dispersion. Although the lethality of weapons through history increased steeply upwards, the daily casualty rates decreased drastically. There must be some reasons for this fact. The most outstanding hypothesis on the subject is that the dispersion of combatants increases more than the lethality of weapons through history. Therefore, that the daily casualty rates decrease accordingly.

The density of troops is the main concept that helps clarify the dispersion in the battlefield as well as on the front line. The lower the density of troops the more dispersion exists in the battlefield. Thus, density is used throughout the chapter for analyses. The battles are in chronological order, so the trends are easy to discern.

Also, the density of the attackers and the defenders are compared in the chapter. The Wilcoxon Signed Ranks Test is applied to each campaign in the chapter to test the hypothesis that the density of the attacker is on average greater than that of the defender.

Proving the hypothesis that the density of the attacker is greater than that of the defender for each campaign is equivalent to the proof of the hypothesis that the attacker's dispersion, which is the inverse of density, is smaller than that of the defender. Also, Kendal's Tau [Ref 3.10] is calculated for overall results to figure out whether dispersion increases with time.

B. DISPERSION AND THE CONCEPT OF DENSITY

Actually, there are two main aspects hidden in the meaning of the concept [Ref 1.6]: the area that is covered by the combatants and the number of combatants covering the area. The change in these two aspects increases or decreases the dispersion of a unit accordingly. Generally, an increase in the area occupied means an increase in the dispersion while an increase in the number of troops causes a decrease in dispersion. A 10000-strong army occupying 10 square kilometers, for example, has a more expansive dispersion than the same amount of troops occupying five square kilometers. Also, a 10,000-strong army occupying ten square kilometers has a smaller dispersion than a 5,000-strong army occupying the same size battlefield.

Understanding the concept that dispersion includes more than the physical area that is occupied clarifies the concept of density to an analyst. A mental picture of dispersion is provided by looking at average troop density on the battlefield in terms of men per square kilometer. The simple formula for the density is:

$$\text{Density} = (\text{Total strength of the army}) / (\text{Total area occupied}) \tag{4.1}$$

$$\text{Dispersion} = 1 / (\text{Density}) \tag{4.2}$$

Obviously, the greater the density (4.1), the smaller the dispersion (4.2). Proving the validity of the hypothesis that the density of troops in the battlefield becomes smaller

in modern battles is equivalent to the proof that dispersion becomes greater in modern warfare. By introducing density to the subject, one sees that both the area that is occupied and the number of troops occupying the area are important.

C. ANALYSIS OF DISPERSION AND DENSITY

1. Napoleonic Wars—(1805-1815)

The Napoleonic Wars were the earliest campaign available for the dispersion analysis. The average density of the campaign is expected to be the highest of all seven campaigns examined, making its dispersion the smallest. The densities of campaign [Table4.1] are 4,494.420 men per square kilometer for the attacker and 3,232 men per square kilometer for the defender.

Note that the troops, being grouped in more compact unit formations with relatively large spaces between units, are not distributed uniformly at this density. The generals of the era, such as Suvarov of Russia [Ref 4.1], were willing to mount massive frontal attacks with highly concentrated troops in the battlefield. This general approach of the commanders is to concentrate the battlefield, especially the front line, causing a small value of dispersion.

To an extent, the effective use of firepower, namely artillery [Ref 4.1], compensated for the increase in density of troops in the battlefield. The possibility that the army of either side in the combat could suffer heavy casualties prevented the commanders from establishing a higher density of troops.

Density (men per km^2) in Napoleonic Wars-1805									
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
AUSTERLITZ	1805	13.000	11.200	85400	73200	2.5	2627.69	2614.29
JENA	1806	9.000	9.000	96000	53000	2.5	4266.67	2355.56
AUERSTADT	1806	6.000	6.000	63500	27000	2.5	4233.33	1800.00
EYLAU	1807	8.000	8.000	78000	80000	2.5	3900.00	4000.00
FRIEDLAND	1807	11.000	11.000	80000	60000	2.5	2909.09	2181.82
VIMIERO	1808	3.200	3.000	13050	19600	2.5	1631.25	2613.33
CORUNNA	1809	5.000	4.000	20600	14800	2.5	1648.00	1480.00
ECKMUEHL	1809	18.000	18.000	74000	66000	2.5	1644.44	1466.67
ASPERN-ES	1809	7.500	5.000	99000	66000	2.5	5280.00	5280.00
THE RAAB	1809	11.500	10.000	35000	37000	2.5	1217.39	1480.00
WAGRAM	1809	24.000	24.000	140000	140000	2.5	2333.33	2333.33
TALAVERA	1809	4.800	4.800	46000	54500	2.5	3833.33	4541.67
BUSSACO	1810	12.000	12.000	65900	51910	2.5	2196.67	1730.33
FUENTES	1811	6.400	6.400	48260	37360	2.5	3016.25	2335.00
ALBUERA	1811	4.800	4.800	23000	30000	2.5	1916.67	2500.00
SALAMANCA	1812	6.400	6.000	46000	42000	2.5	2875.00	2800.00
VITTORIA	1813	11.000	11.000	79062	68024	2.5	2874.98	2473.60
BORODINO	1812	6.000	7.600	120000	120000	2.5	8000.00	6315.79
LUETZEN	1813	8.000	8.000	93000	120000	2.5	4650.00	6000.00
BAUTZEN	1813	9.600	11.200	199000	97000	2.5	8291.67	3464.29
DRESDEN	1813	13.000	13.000	170000	120000	2.5	5230.77	3692.31
LEIPZIG	1813	13.600	20.800	365000	196200	2.5	10735.29	3773.08
HANAU	1813	1.600	3.200	60000	40000	2.5	15000.00	5000.00
LA ROTHIERE	1814	4.800	9.600	110000	40000	2.5	9166.67	1666.67
LAON	1814	6.400	10.400	47600	85000	2.5	2975.00	3269.23
ARCIS-SUR	1814	4.800	4.800	80000	30000	2.5	6666.67	2500.00
LIGNY	1815	12.000	12.000	67567	82895	2.5	2252.23	2763.17
QUATRE BRAS	1815	5.000	5.000	26741	33765	2.5	2139.28	2701.20
WATERLOO	1815	4.000	6.400	68265	137547	2.5	6826.50	8596.69
				Average density (man/km^2)			**4494.42**	**3232.00**

Table 4.1. Density Values Corresponding to Battles of the Napoleonic Wars.

The analysis shows that the average density of the attackers is greater than that of the defenders: 4494.42 and 3232 respectively. However, a hypothesis testing is necessary to statistically support the result. Thus, the Wilcoxon Signed Ranks Test [Ref 3.10] is conducted for the campaign.

The data consist of n observations (x_1,y_1), (x_2,y_2),….., (x_n,y_n) on bivariate random variables, namely density values. The difference between X and Y is calculated as:

$$D_i = X_i - Y_i \quad (4.3)$$

The main idea behind the Wilcoxon Signed Ranks Test is to assess if the expected value of Di, E(Di), is zero. The model is:

H_o: E(Di) = 0 (Average density of attacker = average density of defender)

H_a: E(Di) > 0 (Average density of attacker > average density of defender) (4.4)

As the Wilcoxon Signed Ranks Test is executed for the campaign, the following result is figured out:

```
wilcox.test(NapoleonDensity$DensityA,NapoleonDensity$DensityD,
        alternative="greater", paired=T)
Wilcoxon signed-rank test
data:  NapoleonDensity$DensityA and NapoleonDensity$DensityD
signed-rank normal statistic with correction Z = 1.7195, p-value = 0.0428
alternative hypothesis: true mu is greater than 0
```
(4.5)

The p-value (4.5) of the null hypothesis (4.4) remains below the alpha level of 0.05, so that the null hypothesis is rejected for the campaign. The alternative hypothesis (4.4), which is accepted for the campaign, states that the attacker's density values are greater than defender's density values.

2. American Civil War—(1861-1865)

Obviously, the average density values of either defender or attacker in the American Civil War [Table 4.2] are smaller than those of the Napoleonic Wars [Table 4.1]. This early result supports the hypothesis that the dispersion in the battlefield increases as the density decreases.

One of the main reasons behind the decrease in the density of troops in the period of the American Civil War was the introduction of the conoidal bullet [Ref 1.6], which had a far longer effective range than the spherical ball firing muskets. For practical

purposes, the infantryman's rifle achieved the same effective range as the artilleryman's cannons. In terms of immediate effects upon tactics, and the density of troops, the introduction of the conoidal bullet to the battlefield is one of the most significant changes in weapon lethality.

The hypothesis testing is conducted for the campaign as it is mentioned in the section about Napoleonic Wars. The model (4.4) and the parameters (4.3) are the same as the ones in the Napoleonic War. The result of the Wilcoxon Signed Ranks Test is:

```
wilcox.test(AmeCivilDensity$DensityA,AmeCivilDensity$DensityD,
        alternative="greater", paired=T)
Wilcoxon signed-rank test
data:  AmeCivilDensity$DensityA and AmeCivilDensity$DensityD
signed-rank normal statistic with correction Z = 3.014, p-value = 0.0013        (4.6)
alternative hypothesis: true mu is greater than 0
```

As the p-value (4.6) is smaller than the alpha level of 0.05, the null hypothesis is rejected in favor of the alternative hypothesis stating that the density of the attackers is greater than the density of the defenders.

Density (men per km^2) in American Civil War-1861								
Name of the Battle	Year	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
FIRST BULL RUN	1861	4.50	4.00	35000	32500	3	2592.59	2708.33
WILSON'S CREEK	1861	2.00	2.40	5400	10175	3	900.00	1413.19
BELMONT	1861	1.00	1.00	3144	5000	3	1048.00	1666.67
MILL SPRINGS	1862	0.60	0.80	4000	4000	3	2222.22	1666.67
FORT DONELSON	1862	1.10	1.10	21000	27000	3	6363.64	8181.82
PEA RIDGE	1862	6.00	4.00	16202	10500	3	900.11	875.00
KERNSTOWN	1862	0.80	0.80	3087	7000	3	1286.25	2916.67
SHILOH	1862	6.00	8.00	40355	66812	3	2241.94	2783.83
FRONT ROYAL	1862	2.00	2.00	16000	1063	3	2666.67	177.17
FIRST WINCHESTEI	1862	1.60	1.60	16000	7000	3	3333.33	1458.33
CROSS KEYS	1862	2.80	2.80	10500	5000	3	1250.00	595.24
PORT REPUBLIC	1862	2.40	2.40	15000	3000	3	2083.33	416.67
SEVEN PINES	1862	6.40	6.40	41816	41797	3	2177.92	2176.93
MECHANICSVILLE	1862	3.20	3.20	16808	15631	3	1750.83	1628.23
GAINES'S MILL	1862	3.60	3.60	57018	34214	3	5279.44	3167.96
GLENDALE	1862	3.20	3.20	86748	83345	3	9036.25	8681.77
MALVERN HILL	1862	1.60	3.20	82507	78902	3	17188.96	8218.96
CEDAR MOUNTAIN	1862	1.60	1.60	8030	16848	3	1672.92	3510.00
SECOND BULL RUN	1862	4.00	2.40	75696	48527	3	6308.00	6739.86
SOUTH MOUNTAIN	1862	3.00	2.00	28480	17852	3	3164.44	2975.33
ANTIETAM	1862	6.40	4.00	90000	46000	3	4687.50	3833.33
CORINTH	1862	4.00	3.40	22000	21147	3	1833.33	2073.24
PERRYVILLE	1862	7.20	9.60	36940	16000	3	1710.19	555.56
FREDERICKSBURG	1862	7.80	13.20	106007	72497	3	4530.21	1830.73
MURFREESBORO	1862	13.60	11.20	34732	41400	3	851.27	1232.14
CHANCELLORSVIL	1863	6.80	15.60	113000	60892	3	5539.22	1301.11
CHAMPION'S HILL	1863	4.00	6.80	29373	20000	3	2447.75	980.39
BRANDY STATION	1863	4.40	3.60	12000	10000	3	909.09	925.93
GETTYSBURG	1863	5.10	4.10	75054	83289	3	4905.49	6771.46
CHICKAMAUGA	1863	6.40	7.20	66326	58222	3	3454.48	2695.46
CHATTANOOGA	1863	4.80	7.20	61000	40000	3	4236.11	1851.85
THE WILDERNESS	1864	8.40	6.40	101895	61025	3	4043.45	3178.39
SPOTSYLVANIA	1864	5.20	7.60	90000	50000	3	5769.23	2192.98
NEW MARKET	1864	2.80	2.80	5000	5150	3	595.24	613.10
COLD HARBOR	1864	9.60	11.20	107907	59000	3	3746.77	1755.95
KENESAW MOUNT	1864	15.60	15.60	16225	17733	3	346.69	378.91
PEACHTREE CREEI	1864	1.60	1.60	18832	20139	3	3923.33	4195.63
ATLANTA	1864	4.80	6.40	36934	30477	3	2564.86	1587.34
PETERSBURG	1864	4.00	4.00	63797	41499	3	5316.42	3458.25
GLOBE TAVERN	1864	2.40	2.40	20289	14787	3	2817.92	2053.75
OPEQUON CREEK	1864	3.60	4.80	37711	17103	3	3491.76	1187.71
CEDAR CREEK	1864	4.80	6.40	18410	30829	3	1278.47	1605.68
FRANKLIN	1864	4.00	3.60	26897	27939	3	2241.42	2586.94
NASHVILLE	1864	5.60	8.00	49773	23207	3	2962.68	966.96
BENTONVILLE	1865	2.80	3.60	27000	60000	3	3214.29	5555.56
DINWIDDIE	1865	2.40	2.40	45247	20030	3	6284.31	2781.94
FIVE FORKS	1865	2.40	2.40	30000	10000	3	4166.67	1388.89
SELMA	1865	1.60	8.00	13500	7000	3	2812.50	291.67
SAYLOR'S CREEK	1865	7.20	7.20	30000	21000	3	1388.89	972.22
				Average density (man/km^2)			**3378.29**	**2505.34**

Table 4.2. Density Values Corresponding to Battles of the American Civil War.

3. Franco-Prussian War—(1870-1871)

The Franco–Prussian War also preserves the trend that the density of the battlefield decreases in time. It is noteworthy that the slope of the decrease in the density between the American Civil War and the Franco – Russian War is steeper than before.

The average density values of 2125.886 and 1410.957 [Table 4.3] for the attacker and the defender respectively imply that an attacking soldier occupies 470.4 square meters while a defending soldier occupies 708.74 square meters.

A discriminating feature of the campaign from the other wars is that the battles in the campaign were fought on a relatively narrow front line [Ref 3.3] that made the troop concentrations impossible for both sides. This was mainly due to features of the terrain as well as the weather. For example, in the battle of Sedan, which ended in an overwhelming Prussian victory over the French army, both sides engaged each other only in battalion size of units since the terrain and extremely heavy fog did not allow either side to concentrate their troops in the battlefield as much as they wanted. Thus, the density of the troops in the battlefield dramatically decreased.

Since the battlefield terrain [Ref 3.3] did not let commanders of either side to concentrate their troops, they generally split their armies into small groups to attack the enemy from other positions. This tactic decreased troop density in a single battle, yet increased the number of battles fought in the campaign.

Density (men per km^2) in Franco-Prussian War-1870								
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
WEISSENBURG	1870	5	2.5	51000	6000	4	2550.00	600.00
FROESCHWILLER	1870	10	8	82000	41000	4	2050.00	1281.25
SPICHERN	1870	6	2.5	42000	28000	4	1750.00	2800.00
MARS LA TOUR	1870	12	10	91000	113000	4	1895.83	2825.00
GRAVELOTTE	1870	16	16	187000	113000	4	2921.88	1765.63
SEDAN	1870	13	13	200000	120000	4	3846.15	2307.69
COULMIERS	1870	10	10	60000	20000	4	1500.00	500.00
ORLEANS	1870	60	60	86000	116000	4	358.33	483.33
LE MANS	1871	25	25	72000	88000	4	720.00	880.00
BELFORT	1871	7.5	15	110000	40000	4	3666.67	666.67
				Average density (men/km^2)			**2125.89**	**1410.96**

Table 4.3. Density Values Corresponding to Battles of the Franco-Prussian War.

When the Wilcoxon Signed Ranks Test is applied to the Franco-Prussian War with the model (4.4) and the parameters (4.3) previously stated, the following result is calculated:

```
wilcox.test(FrancoDensity$DensityA,FrancoDensity$DensityD,alternative="greater", paired=T)

Exact Wilcoxon signed-rank test

data: FrancoDensity$DensityA and FrancoDensity$DensityD

signed-rank statistic V = 38, n = 9, p-value = 0.0371                                    (4.7)

alternative hypothesis: true mu is greater than 0
```

The p-value (4.7) of the model corresponding to the Franco-Prussian War is less than the alpha level of 0.05. Thus, the null hypothesis is rejected in favor of the alternative hypothesis, which states that the density of the attacker is greater than the density of the defender.

4. World War I (WWI)—(1914-1918)

In the 20th Century, the average space occupied by each soldier on a battlefield has increased steadily, so the dispersion has increased dramatically in WWI. Being the first major campaign in the 20th Century, WWI also preserves the trend that as the density in the battlefield decreases an increase in the dispersion is revealed.

On the average, 186.745 defenders fight 345.1670 attackers [Table 4.4] in a square kilometer in the campaign. That the density decreased below a three-digit number in WWI is an important point to mention.

The analysis of the way in which WWI was conducted reveals the reasons for the decreased density of troops in the battlefield, although one would expect the density of combatants to increase in such a huge campaign. First of these reasons, WWI covered most of Europe, the Middle East, Northern Africa, and Manchuria [Ref 4.1]. This extended the width of the front line beyond the number of combatants the nations summon. The campaign was full of trench battles that might have allowed the opposing forces to concentrate their forces, and increase the density of troops, but they lacked an adequate number of combatants. This decreased the overall density.

Partially mentioned above, the lack of human resource [Ref 4.1] to supply the battlefield with an ample amount of troops decreased the density of ground troops. Since the combating nations of WWI had engaged in numerous campaigns prior to WWI, they did not possess an abundance of soldiers. Thus, this insufficiency of troops decreased the density.

The Wilcoxon Signed Ranks Test is applied to the campaign to test the hypothesis if the density of the attacker was greater than that of the defender. The model (4.4) and the parameters (4.3) were mentioned before. The results of the test are:

```
wilcox.test(WW.IDensity$DensityA,WW.IDensity$Density, alternative="greater", paired=T)

Wilcoxon signed-rank test

data: WW.IDensity$DensityA and WW.IDensity$DensityD

signed-rank normal statistic with correction Z = 8.3763, p-value = 0

alternative hypothesis: true mu is greater than 0
```

(4.8)

The resulting p-value (4.8), which is smaller than the alpha level of 0.05, indicates that the null hypothesis of the model is rejected in favor of the alternative hypothesis. Thus, it is concluded that density of the attacker was greater than that of the defender.

Density (men per km^2) in World War I-1914								
Name of the Battle	Year	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
ALSACE-LORRAINE I	1914	225	225	457000	345000	17.33	117.20	88.48
ALSACE-LORRAINE II	1914	225	225	350000	400000	17.33	89.76	102.58
THE ARDENNES	1914	100	100	360000	400000	17.33	207.73	230.81
THE SAMBRE	1914	53	53	440000	254000	17.33	479.05	276.54
MONS	1914	35	35	260000	70000	17.33	428.65	115.41
LE CATEAU	1914	23	23	250000	40000	17.33	627.21	100.35
GUISE	1914	50	50	260000	200000	17.33	300.06	230.81
HEIGHTS OF NANCY	1914	144	144	350000	276000	17.33	140.25	110.60
OURCQ I	1914	18	18	100000	45000	17.33	320.57	144.26
OURCQ II	1914	32	32	198000	157000	17.33	357.04	283.11
PETIT MORIN	1914	37	27.8	227000	82000	17.33	354.02	170.20
TWO MORINS	1914	30.4	30.4	90000	13000	17.33	170.83	24.68
MARSHES OF ST.GOND	1914	42	42	101000	141000	17.33	138.76	193.72
VITRY LE FRANCOIS	1914	64	64	113000	170000	17.33	101.88	153.27
GAP OF REVIGNY	1914	64	64	142000	180000	17.33	128.03	162.29
THE AISNE	1914	112	112	343000	290000	17.33	176.72	149.41
EASTERN CHAM	1915	17	17	163182	85220	17.33	553.89	289.26
NEUVE CHAPELLE	1915	3.6	3.6	87000	40000	17.33	1394.50	641.15
YPRES II	1915	23	23	150000	190000	17.33	376.33	476.68
FESTUBERT	1915	6	6	90365	30000	17.33	869.06	288.52
LOOS	1915	7.2	7.2	298437	75000	17.33	2391.78	601.08
STALLUPONEN	1914	32	32	50000	40000	17.33	90.16	72.13
GUMBINNEN	1914	64	64	120000	150000	17.33	108.19	135.24
TANNENBERG	1914	120	120	187000	160000	17.33	89.92	76.94
MASURIAN LAKES	1914	121	121	288600	273000	17.33	137.63	130.19
KRASNIK	1914	64	64	350000	260000	17.33	315.57	234.42
KOMAROV	1914	72	72	300000	260000	17.33	240.43	208.37
GNILA LIPA	1914	150	150	240000	480000	17.33	92.33	184.65
RAVA RUSSKA	1914	160	160	900000	936000	17.33	324.58	337.56
LODZ	1914	120	120	260000	400000	17.33	125.02	192.34
THE JADAR	1914	74	74	200000	200000	17.33	155.96	155.96
THE KOLUBRA	1914	106	106	200000	300000	17.33	108.87	163.31
WINTER BATTLE	1915	209	209	650000	300000	17.33	179.46	82.83
GOLICE-TARNOW	1915	121	121	216000	219000	17.33	103.01	104.44
FIRST ISONZO	1915	16	16	200000	100000	17.33	721.29	360.65
SECOND ISONZO	1915	29	29	200000	128500	17.33	397.95	255.69
THIRD ISONZO	1915	29	29	356000	157000	17.33	708.36	312.39
FOURTH ISONZO	1915	29	29	311000	136000	17.33	618.82	270.61
FIRST DARDANELLES	1915	11.2	11.2	32000	10000	17.33	164.87	51.52
SUVLA BAY	1915	4.8	4.8	25000	15800	17.33	300.54	189.94
KUT-EL-AMARA	1915	16	16	11000	11300	17.33	39.67	40.75
CTESIPHON	1915	12.1	16	13756	20400	17.33	65.60	73.57
FIRST SOMME	1916	62.4	62.4	600000	300000	17.33	554.84	277.42
SOMME-FOURTH	1916	19.2	19.2	290000	95000	17.33	871.56	285.51
SOMME-OVILLERS	1916	1.4	1.4	11300	2800	17.33	465.75	115.41
SOMME-BAZENTIN	1916	4.5	4.5	45000	15000	17.33	577.03	192.34
SOMME-FLERS	1916	9.600001	9.600001	190000	90000	17.33	1142.05	540.97
CAUCASUS WINTER	1916	110	110	103000	61000	17.33	54.03	32.00
LAKE NAROTCH	1916	20	20	350000	180000	17.33	1009.81	519.33
1916 BRUSILOV	1916	400	400	600000	500000	17.33	86.56	72.13
FIFTH ISONZO	1916	48	48	300000	160000	17.33	360.65	192.34
ASIAGO	1916	48	48	213000	118000	17.33	256.06	141.85
TRENTINO COUNTER	1916	48	48	200000	172000	17.33	240.43	206.77
SIXTH ISONZO	1916	22.5	22.5	308000	168000	17.33	789.90	430.85
ARRAS	1917	32	32	276000	120000	17.33	497.69	216.39
AISNE II	1917	64	64	1000000	480000	17.33	901.62	432.78
MESSINES	1917	19	19	180000	100000	17.33	546.66	303.70
YPRES III	1917	48	48	380000	200000	17.33	456.82	240.43
CAMBRAI I	1917	16	16	90000	75000	17.33	324.58	270.48
CAMBRAI II	1917	22	22	130000	90000	17.33	340.97	236.06
TENTH ISONZO	1917	25	25	280000	165000	17.33	646.28	380.84
ELEVENTH ISONZO	1917	30	30	518000	252000	17.33	996.35	484.71
CAPORETTO	1917	160	160	602000	574000	17.33	217.11	207.01
TIGRIS CROSSING	1917	7.2	40	46000	10500	17.33	368.66	15.15
GAZA I	1917	12	19.2	25000	26000	17.33	120.22	78.14
GAZA II	1917	24	24	25000	20000	17.33	60.11	48.09
GAZA III	1917	32	48	72000	34000	17.33	129.83	40.87
JUNCTION STATION	1917	24	24	85000	15500	17.33	204.37	37.27
SECOND SOMME- I	1918	97	97	800000	400000	17.33	475.90	237.95
SECOND SOMME- II	1918	140	140	700000	600000	17.33	288.52	247.30
LYS	1918	32	32	500000	400000	17.33	901.62	721.29
YVONNE & ODETTE	1918	0.3	0.3	3072	650	17.33	590.88	125.02

Table 4.4. Density Values Corresponding to Battles of WWI.

Density (men per km^2) in World War I-1914 (Continued)								
Name of the Battle	Year	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
CHEMIN-DES-DAMES	1918	56.3	56.3	250000	75000	17.33	256.23	76.87
CANTIGNY	1918	1.5	1.5	8679	725	17.33	333.87	27.89
BELLEAU WOOD	1918	4.4	4.4	9437	6436	17.33	123.76	84.40
HILL 142	1918	0.3	0.3	2913	2458	17.33	560.30	472.78
WEST WOOD I	1918	0.8	0.8	1740	1121	17.33	125.50	80.86
BOURESCHES I	1918	0.7	0.7	2733	1352	17.33	225.29	111.45
HILL 192	1918	1.6	1.6	3608	3955	17.33	130.12	142.64
WEST WOOD II	1918	0.8	0.8	3343	1798	17.33	241.13	129.69
NORTH WOOD I	1918	0.8	0.8	1747	1952	17.33	126.01	140.80
BOURESCHES II	1918	1.6	1.6	3690	2629	17.33	133.08	94.81
NORTH WOOD II	1918	1.2	1.2	1697	1428	17.33	81.60	68.67
NORTH WOOD III	1918	1.2	1.2	1256	1565	17.33	60.40	75.25
NORTH WOOD IV	1918	0.8	0.8	4453	1546	17.33	321.19	111.51
VAUX	1918	1.8	1.8	12812	10358	17.33	410.72	332.05
LA ROCHE WOOD	1918	0.7	1	4515	5182	17.33	372.19	299.02
LA ROCHE WOOD	1918	0.7	0.7	4508	5177	17.33	371.61	426.76
NOYON-MONTDIDIER	1918	43	43	275000	300000	17.33	369.03	402.58
CHAMPAGNE-MARNE	1918	105	105	400000	450000	17.33	219.82	247.30
AISNE-MARNE I	1918	130	130	750000	450000	17.33	332.90	199.74
MISSY AUX BOIS	1918	0.7	0.7	5004	3013	17.33	412.50	248.37
BREUIL	1918	0.8	0.8	5039	2663	17.33	363.46	192.08
ST. AMAND FARM	1918	0.3	0.3	1150	400	17.33	221.20	76.94
BEAUREPAIRE FARM	1918	1.3	1.3	4480	565	17.33	198.85	25.08
CRAVANCON FERME	1918	1.5	1.5	10345	2420	17.33	397.96	93.09
CHAUDUN	1918	0.6	0.6	1611	800	17.33	154.93	76.94
AISNE-MARNE II	1918	128	128	725000	400000	17.33	326.84	180.32
BERZY LE SEC	1918	1	1	4000	350	17.33	230.81	20.20
BUZANCY RIDGE	1918	1	1	5300	554	17.33	305.83	31.97
PICARDY 1918, I	1918	70	70	225000	170000	17.33	185.48	140.14
PICARDY 1918, II	1918	150	150	300000	200000	17.33	115.41	76.94
ST. MIHIEL	1918	72	72	400000	100000	17.33	320.57	80.14
LAHAYVILLE-BOIS	1918	1.2	1.2	13208	2090	17.33	635.12	100.50
MEUSE-ARGONNE I	1918	53	53	300000	190000	17.33	326.62	206.86
BLANC MONT I	1918	6.4	6.4	26000	13000	17.33	234.42	117.21
MEDEAH FARM	1918	0.6	0.6	1921	155	17.33	184.75	14.91
ESSEN HOOK	1918	0.8	0.8	1420	216	17.33	102.42	15.58
BLANC MONT RIDGE	1918	1.3	1.3	1400	458	17.33	62.14	20.33
SOMMEPY WOOD	1918	1.5	1.5	9230	670	17.33	355.07	25.77
BLANC MONT II	1918	4	4	18000	10000	17.33	259.67	144.26
MEUSE-ARGONNE II	1918	104	104	500000	300000	17.33	277.42	166.45
EXERMONT-MONTREF	1918	0.9	0.9	5336	3245	17.33	342.12	208.05
MAYACHE RAVINE	1918	1	1	5427	1899	17.33	313.16	109.58
LA NEUVILLE	1918	1	1	5365	1940	17.33	309.58	111.94
FERME DES GRANGES	1918	1	1	5461	2587	17.33	315.12	149.28
HILL 212	1918	0.8	0.8	5022	3335	17.33	362.23	240.55
BOIS DE BOYON	1918	0.9	0.9	4778	2925	17.33	306.34	187.54
HILL 272	1918	0.9	0.9	2950	2563	17.33	189.14	164.33
MEUSE-ARGONNE III	1918	169	169	600000	380140	17.33	204.86	129.80
REMILLY-AILLICOURT	1918	0.5	0.5	1210	296	17.33	139.64	34.16
HILL 252-PONT MAUGIS	1918	1.1	1.1	1989	1655	17.33	104.34	86.82
THE PIAVE	1918	144	144	840000	784000	17.33	336.60	314.16
MEGIDDO	1918	24	24	51170	18250	17.33	123.03	43.88
				Average density (man/km^2)			**345.17**	**186.75**

Table 4.4. (continued) Density Values Corresponding to Battles of WWI.

5. World War II (WWII)—(1939-1945)

WWII as a campaign is assumed to be the very first of modern warfare in terms of weapons, tactics that have been introduced to the battlefields. Also in WWII, the density of weapon lethality suddenly rose via unprecedented heavy firepower in the battlefield.

Tacticians have responded to the abrupt change in the lethality of weapons [Ref 4.1] by increasing the dispersion of the armies to minimize the effect of such a devastating arsenal. Also, WWII was the first group of battles in which the density of troops reached two-digit numbers: 72.584 for the attacker and 30.902 for the defender [Table 4.5].

WWII created a new era of warfare. As the most devastating of all campaigns in history, WWII involved most of the world. Also in WWII, the advent of a nuclear arsenal appeared at the very end of the campaign.

The massive use of firepower [Ref 4.1] by air, naval, or ground forces abruptly decreased the troop density in the battlefield. Firepower, on the other hand, evolved as the main factor in modern warfare. The generals preferred massive firepower instead of bloody infantry attacks. This drastic change in the conduct of war rapidly decreased troop density in the battlefield.

Density (men per km^2) in World War II-1944								
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
CAMBRAI	1940	90	90	124000	120000	57	24.17	23.39
ARRAS	1940	61	61	220476	105223	57	63.41	30.26
BOOS	1940	61	61	220476	105223	57	63.41	30.26
JITRA	1941	61	61	214336	101528	57	61.64	29.20
ROVNO	1941	61	61	211000	97000	57	60.68	27.90
MOSCOW'S DEFENSE	1941	0.6	1.6	465	188	57	13.60	2.06
MOSCOW COUNTER	1941	25	25	10300	22019	57	7.23	15.45
ALAM HALFA	1942	32	32	24098	5000	57	13.21	2.74
EL ALAMEIN II	1942	13	13	12917	4250	57	17.43	5.74
OPERATION LIGHT	1942	6	6	12917	4250	57	37.77	12.43
ALAMEIN BRIDGE	1942	11	11	12447	8390	57	19.85	13.38
OPERATION SUPER	1942	4.8	4.8	14730	11230	57	53.84	41.05
CHOUIGUI PASS	1942	14.5	14.5	15000	12917	57	18.15	15.63
POGORELOYE GOROD	1942	9.7	9.7	14733	12691	57	26.65	22.95
EL GUETTAR	1943	12	12	14730	6995	57	21.54	10.23
SEDJANNE-BIZERTE	1943	7.5	7.5	15576	6702	57	36.44	15.68
AMPHITHEATER	1943	14.5	14.5	13300	18912	57	16.09	22.88
PORT OF SALERNO	1943	1.5	1.5	14557	8068	57	170.26	94.36
SELE-CALORE	1943	9	9	18210	6435	57	35.50	12.54
BATTIPAGLIA I	1943	2	2	16857	8000	57	147.87	70.18
VIETRI	1943	9.60	9.60	17765	8158	57	32.47	14.91
TOBACCO FACTORY	1943	8	8	21265	6435	57	46.63	14.11
BATTIPAGLIA II	1943	9	9	18476	7250	57	36.02	14.13
EBOLI	1943	5.2	5.2	17034	5152	57	57.47	17.38
VIETRI II	1943	9	9	14600	8138	57	28.46	15.86
GRAZZANISE	1943	9	9	16400	7239	57	31.97	14.11
CAIAZZO	1943	9	9	17500	8128	57	34.11	15.84
CAPUA	1943	6.4	6.4	14000	8088	57	38.38	22.17
CASTEL VOLTURNO	1943	4.8	4.8	16870	6321	57	61.66	23.10
MONTE ACERO	1943	5	5	19513	6750	57	68.47	23.68
TRIFLISCO	1943	6	6	16600	6566	57	48.54	19.20
DRAGONI	1943	8	8	17404	6566	57	38.17	14.40
CANAL I	1943	0.5	0.5	7942	5200	57	278.67	182.46
MONTE GRANDE	1943	4.5	4.5	16350	7942	57	63.74	30.96
CANAL II	1943	2	2	17765	7588	57	155.83	66.56
FRANCOLISE	1943	2	2	20744	3288	57	181.96	28.84
SANTA MARIA	1943	1	1	5551	3288	57	97.39	57.68
MONTE CAMINO I	1943	7	7	19350	6750	57	48.50	16.92
MONTE LUNGO	1943	7	7	15317	17976	57	38.39	45.05
POZZILLI	1943	11	11	17766	15098	57	28.33	24.08
MONTE CAMINO II	1943	11	11	26029	9834	57	41.51	15.68
MONTE ROTONDO	1943	8	8	26490	4515	57	58.09	9.90
CALABRITTO	1943	3.2	3.2	7418	5000	57	40.67	27.41
MONTE CAMINO III	1943	2	2	27518	17730	57	241.39	155.53
MONTE MAGGIORE	1943	2	2	13400	7077	57	117.54	62.08
LENINGRAD	1943	9.60	9.60	41974	20496	57	76.71	37.46
OBOYAN-KURSK	1943	4	4	21478	9761	57	94.20	42.81
OPERATION CITADEL	1943	8	8	15637	19613	57	34.29	43.01
OBOYAN-KURSK	1943	7.5	7.5	18702	9250	57	43.75	21.64
OBOYAN-KURSK	1943	5.5	5.5	17970	8141	57	57.32	25.97
PROKHOROVKA	1943	5	5	16458	7500	57	57.75	26.32
KURSK COUNTER.	1943	5.5	5.5	18308	8215	57	58.40	26.20
BELGOROD	1943	4	4	23190	7627	57	101.71	33.45
MELITOPOL	1943	9	9	13095	4563	57	25.53	8.89
TARAWA-BETIO	1943	6	6	17912	6653	57	52.37	19.45
SIDI BOU ZID I	1943	15	15	18030	6653	57	21.09	7.78
SIDI BOU ZID II	1943	8.5	8.5	17345	12569	57	35.80	25.94
KASSERINE PASS	1943	6	6	17313	11343	57	50.62	33.17
APRILIA I	1944	5.6	5.6	22374	12815	57	70.09	40.15
THE FACTORY	1944	7.8	7.8	19971	11928	57	44.92	26.83
CAMPOLEONE	1944	14	14	17925	6957	57	22.46	8.72
CAMPOLEONE	1944	14	14	20683	12327	57	25.92	15.45
CARROCETO	1944	6.5	6.5	19047	10593	57	51.41	28.59
MOLETTA RIVER	1944	5	5	18000	13715	57	63.16	48.12
APRILIA II	1944	9	9	15557	7659	57	30.33	14.93
FACTORY COUNTER	1944	11	11	29711	15801	57	47.39	25.20
BOWLING ALLEY	1944	3	3	17300	6108	57	101.17	35.72
MOLETTA RIVER II	1944	5	5	22641	13012	57	79.44	45.66
FIOCCIA	1944	4	4	23604	19255	57	103.53	84.45
SANTA MARIA	1944	5.5	5.5	26607	10111	57	84.87	32.25
SAN MARTINO	1944	7	7	38011	10855	57	95.27	27.21
CASTELLONORATO	1944	4	4	15721	3700	57	68.95	16.23

Table 4.5. Density Values Corresponding to Battles of WWII.

Density (men per km^2) in World War II-1944 (Continued)								
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
SPIGNO	1944	12	12	18228	7500	57	26.65	10.96
FORMIA	1944	21	21	76213	57500	57	63.67	48.04
MONTE GRANDE	1944	11	11	126000	30700	57	200.96	48.96
ITRI-FONDI	1944	9	9	25497	27673	57	49.70	53.94
TERRACINA	1944	5	5	15646	8325	57	54.90	29.21
MOLETTA OFFENSIVE	1944	5	5	17232	6000	57	60.46	21.05
ANZIO-ALBANO	1944	45	45	40619	15000	57	15.84	5.85
ANZIO BREAKOUT	1944	25	25	59631	41500	57	41.85	29.12
CISTERNA	1944	25	25	60794	39580	57	42.66	27.78
SEZZE	1944	10.4	10.4	7500	4800	57	12.65	8.10
VELLETRI	1944	12.5	12.5	32283	19632	57	45.31	27.55
CAMPOLEONE	1944	9.5	9.5	20493	20250	57	37.84	37.40
VILLA CROCETTA	1944	48	48	99583	23588	57	36.40	8.62
ARDEA	1944	16.6	16.6	43587	11185	57	46.07	11.82
FOSSO DI CAMPOL	1944	7.7	7.7	25881	7555	57	58.97	17.21
LANUVIO	1944	49	49	92393	28382	57	33.08	10.16
LARIANO	1944	11.3	11.3	10348	6519	57	16.07	10.12
VIA ANZIATE	1944	64	64	88941	32396	57	24.38	8.88
VALMONTONE	1944	3.5	3.5	7935	5366	57	39.77	26.90
TARTO-TIBER	1944	11.3	11.3	15871	6999	57	24.64	10.87
IL GIOGIO PASS	1944	11.3	11.3	16232	6713	57	25.20	10.42
ST. LO	1944	51.2	51.2	90078	30712	57	30.87	10.52
OPERATION GOOD	1944	4.5	4.5	19773	6044	57	77.09	23.56
OPERATION COBRA	1944	32	32	89977	31501	57	49.33	17.27
MORTAIN	1944	3.6	3.6	15224	5044	57	74.19	24.58
CHARTRES	1944	14.5	14.5	10000	8634	57	12.10	10.45
MELUN	1944	12	12	87000	19996	57	127.19	29.23
SEINE RIVER	1944	12	12	36678	4849	57	53.62	7.09
MOSELLE-METZ	1944	9	9	48000	60000	57	93.57	116.96
METZ	1944	8	8	7000	12000	57	15.35	26.32
ARRACOURT	1944	65	65	132000	150000	57	35.63	40.49
WESTWALL	1944	700	700	1100000	1372000	57	27.57	34.39
SCHMIDT	1944	1060	1060	1060300	880000	57	17.55	14.56
SEILLE-NIED	1944	36	36	54180	12035	57	26.40	5.87
FORET DE CHATEAU	1944	13	13	120000	30000	57	161.94	40.49
MORHANGE	1944	16	16	62000	45000	57	67.98	49.34
MORHANGE-FAUL	1944	30	30	140000	75000	57	81.87	43.86
BOURGALTROFF	1944	20	20	60000	149000	57	52.63	130.70
SARRE-ST. AVOLD	1944	25	25	56000	129000	57	39.30	90.53
BAERENDORF I	1944	25	25	78000	82300	57	54.74	57.75
BAERENDORF II	1944	250	250	980600	280000	57	68.81	19.65
BURBACH-DURSTEL	1944	16	16	70000	15000	57	76.75	16.45
DURSTEL-FAERBER	1944	111	111	524724	210000	57	82.93	33.19
SARRE-UNION	1944	180	180	254950	84500	57	24.85	8.24
SARRE-SINGLING	1944	12	12	25100	8230	57	36.70	12.03
SINGLING-BINING	1944	25	25	397607	72000	57	279.02	50.53
SAUER RIVER	1944	10	10	16100	8500	57	28.25	14.91
ST. VITH	1944	440	440	1200000	900000	57	47.85	35.89
BASTOGNE	1944	5.5	5.5	39000	3300	57	124.40	10.53
KORSUN-SCHEVCHEN	1944	7	7	38500	12900	57	96.49	32.33
NIKOPOL BRIDGE	1944	10	10	12700	5100	57	22.28	8.95
SEVASTOPOL	1944	12	12	17550	6400	57	25.66	9.36
BEREZINA RIVER	1944	590	590	1250000	800000	57	37.17	23.79
LVOV-SANDOMIERZ	1944	480	480	2200000	560000	57	80.41	20.47
BRODY (PHASE I)	1944	500	500	1220000	780000	57	42.81	27.37
BRODY (PHASE II)	1944	2	2	10800	3100	57	94.74	27.19
VISTULA RIVER	1944	2.5	2.5	12115	3900	57	85.02	27.37
VISTULA RIVER	1944	2	2	13600	3710	57	119.30	32.54
YASSY-KISHINEV	1944	12	12	147000	75000	57	214.91	109.65
RAPIDO NORTH I	1944	1.2	1.2	9000	4836	57	131.58	70.70
RAPIDO NORTH II	1944	4.7	4.7	33915	18300	57	126.60	68.31
RAPIDO SOUTH I	1944	0.8	0.8	3200	1600	57	70.18	35.09
RAPIDO SOUTH II	1944	1.8	1.8	32000	2685	57	311.89	26.17
BOWLING ALLEY I	1944	1.6	1.6	22888	1400	57	250.96	15.35
BOWLING ALLEY II	1944	2.2	2.2	18398	2900	57	146.71	23.13
BOWLING ALLEY III	1944	2.6	2.6	18111	4731	57	122.21	31.92
MORTAIN I	1944	3.4	3.4	16291	2600	57	84.06	13.42
MORTAIN II	1944	3	3	14594	5000	57	85.35	29.24
SCHMIDT I	1944	3	3	15986	4500	57	93.49	26.32
SCHMIDT II	1944	3	3	15764	4050	57	92.19	23.68
SCHMIDT III	1944	1.8	1.8	6850	15350	57	66.76	149.61
WAHLERSCHEID	1944	2.2	2.2	15109	5140	57	120.49	40.99

Table 4.5. (continued) Density Values Corresponding to Battles of WWII.

Density (men per km2) in World War II-1944 (Continued)									
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
KRINKELT-ROCHERATH	1944	3.6	3.6	16043	3338	57	78.18	16.27
KRINKELT-ROCHERATH	1944	3.6	3.6	4000	15777	57	19.49	76.89
SCHNEE EIFEL CENTE	1944	4	4	15840	3000	57	69.47	13.16
SCHNEE EIFEL SOUTH	1944	4	4	15205	2600	57	66.69	11.40
SCHNEE EIFEL NORTH	1944	2.5	2.5	16091	3500	57	112.92	24.56
SCHNEE EIFEL NORTH	1944	2	2	16002	2500	57	140.37	21.93
OUR RIVER CENTER	1944	1.5	1.5	5237	2500	57	61.25	29.24
TARGUL FRUMOS	1944	3	3	15808	2000	57	92.44	11.70
TARTO-TIBER	1944	5	5	19082	2000	57	66.95	7.02
VISTULA-ODER	1945	4	4	18388	2900	57	80.65	12.72
EAST PRUSSIA	1945	3.8	3.8	21247	3000	57	98.09	13.85
CIECHANOW (PHASE I)	1945	2.3	2.3	17163	3000	57	130.92	22.88
CIECHANOW (PHASE II	1945	2.1	2.1	18095	3900	57	151.17	32.58
SEELOW HEIGHTS	1945	2.3	2.3	19714	5284	57	150.37	40.31
MUTANKIANG	1945	2.5	2.5	20973	4757	57	147.18	33.38
IWO JIMA	1945	2.9	2.9	19658	4227	57	118.92	25.57
IWO JIMA - SURIBACHI	1945	3	3	18777	4000	57	109.81	23.39
IWO JIMA - FINAL	1945	3	3	18660	4250	57	109.12	24.85
BEACHHEAD	1945	3	3	19047	3250	57	111.39	19.01
OUTPOSTS	1945	22	22	6,400	5,333	57	5.10	4.25
TOMB HILL-OUKI	1945	4	4	2,738	8,380	57	12.01	36.75
SKYLINE RIDGE-ROCK	1945	4	4	7,000	5,303	57	30.70	23.26
KOCHI RIDGE-ONAGA I	1945	1	1	8,000	2,200	57	140.35	38.60
KOCHI RIDGE-ONAGA I	1945	1	1	7,600	2,200	57	133.33	38.60
KOCHI RIDGE-ONAGA I	1945	1	1	7,700	1,800	57	135.09	31.58
JAPANESE COUNTER	1945	1	1	7,538	1,800	57	132.25	31.58
KOCHI RIDGE IV	1945	2	2	14,600	4,500	57	128.07	39.47
SHURI (PHASE I)	1945	2.4	2.4	15,736	5,050	57	115.03	36.92
JAPANESE COUNTER	1945	2.6	2.6	10,000	4,625	57	67.48	31.21
SHURI (PHASE II)	1945	2	2	8,150	3,700	57	71.49	32.46
SHURI (PHASE III)	1945	3	3	8,500	4,600	57	49.71	26.90
HILL 95-I	1945	1.7	1.7	6,200	5,025	57	63.98	51.86
HILL 95-II	1945	2	2	4,350	3,450	57	38.16	30.26
HILL 95-III	1945	1.2	1.2	4,950	3,700	57	72.37	54.09
YAEJU-DAKE	1945	1	1	8,300	1,400	57	145.61	24.56
HILLS 153 AND 115	1945	2.5	2.5	3,300	1,357	57	23.16	9.52
ADVANCE	1945	3	3	9,100	6,600	57	53.22	38.60
ADVANCE TO SHURI	1945	1	1	4,100	3,900	57	71.93	68.42
KAKAZU AND TOMBS	1945	9	9	11,000	4,300	57	21.44	8.38
NISHIBARU RIDGE	1945	6	6	14,300	2,050	57	41.81	5.99
MAEDA ESCARPMENT	1945	1	1	12,800	4,150	57	224.56	72.81
ATTACK ON SHURI	1945	12	12	43,800	5,340	57	64.04	7.81
ATTACK ON SHURI	1945	19		35,170		57	32.47	
ATTACK ON SHURI	1945	7		38,011		57	95.27	
ADVANCE TO YUZA	1945	1.8		17,000		57	165.69	
ATTACK ON YUZA	1945	5		11,821		57	41.48	
CAPTURE OF YUZA	1945	0.82		189		57	4.04	
				Average density (men/km^2)			72.58	30.90

Table 4.5. (continued) Density Values Corresponding to Battles of WWII.

The model (4.4) and the parameters (4.3) for the hypothesis testing to determine whether the attacker's density was greater than the defender's density were mentioned before. The results of the Wilcoxon Signed Ranks Test are:

```
wilcox.test(WW.IIDensity$DensityA,WW.IIDensity$DensityD,alternative="greater", paired=T)
Wilcoxon signed-rank test
data: WW.IIDensity$DensityA and WW.IIDensity$DensityD
signed-rank normal statistic with correction Z = 10.6537, p-value = 0          . (4.9)
alternative hypothesis: true mu is greater than 0
```

The p-value (4.9) of the model is almost zero, far smaller than the alpha level of 0.05. Thus, the null hypothesis is rejected in favor of the alternative, claiming that the density of the attacker is greater than the density of the defender.

6. Arab–Israel War—1973

The Arab–Israel War in 1973 is the last group of battles that is analyzed for the concept of dispersion. The campaign preserves the general trend that the density of troops in the battlefield decreases as the dispersion increases. The campaign fits to the general trend, yet one important fact surfaces as the absolute value of the slope decreases for both the attacker and the defender. The average density values of 30.031 and 29.928 [Table 4.6] are similar to those of WWII.

Dispersion (men per km^2) in Arab-Israel-1973									
Name of the Battle	Time	The front line width of the attacker LA(km)	The front line width of the defender LD(km)	The total strength of the attacker A	The total strength of the defender D	Average depth of the battle c(km)	Density of the attacker A / (LA*c)	Density of the defender D / (LD*c)
KANTARA-FIRDAN	1973	27	27	25850	67440	57	16.80	43.82
EGYPTIAN OFFENS	1973	50	50	81160	43400	57	28.48	15.23
EGYPTIAN OFFENS	1973	50	50	57960	28600	57	20.34	10.04
DEVERSOIR	1973	14	14	22790	30970	57	28.56	38.81
DEVERSOIR	1973	11	11	28900	36840	57	46.09	58.76
DEVERSOIR WEST	1973	11	11	19600	18180	57	31.26	29.00
ISMAILIA	1973	20	20	17000	23860	57	14.91	20.93
JEBEL GENEIFA	1973	18	18	16200	35633	57	15.79	34.73
SHALLUFA I	1973	32	32	16200	25600	57	8.88	14.04
SHALLUFA II	1973	32	32	11700	22570	57	6.41	12.37
SUEZ	1973	6	6	14681	22570	57	42.93	65.99
ADABIYA	1973	13	13	10900	14620	57	14.71	19.73
KUNEITRA	1973	15	15	17750	3630	57	20.76	4.25
AHMADIYEH	1973	7.5	7.5	22750	5745	57	53.22	13.44
RAFID	1973	14	14	19525	4958	57	24.47	6.21
YEHUDA-EL AL	1973	12.5	20	21984	6300	57	30.85	5.53
NAFEKH	1973	7	7	12500	6946	57	31.33	17.41
TEL FARRIS	1973	14	14	17833	23750	57	22.35	29.76
HUSHNIYAH	1973	12	20	12733	14683	57	18.62	12.88
MOUNT HERMONIT	1973	7.5	7.5	31650	5395	57	74.04	12.62
MOUNT HERMON I	1973	1	1	2692	1583	57	47.23	27.77
TEL SHAMS	1973	5	5	16100	19400	57	56.49	68.07
TEL SHAAR	1973	3	3	14700	21500	57	85.96	125.73
TEL EL HARA	1973	12	20	12500	14300	57	18.27	12.54
KFAR SHAMS	1973	20	20	11000	12000	57	9.65	10.53
NABA	1973	9	9	11500	11000	57	22.42	21.44
ARAB COUNTER	1973	25	25	35750	16100	57	25.09	11.30
MOUNT HERMON II	1973	1	1	5,700	4,750	57	100.00	83.33
MOUNT HERMON III	1973	2	2	11,400	4,750	57	100.00	41.67
				Average dispersion (men/km^2)			**35.03**	**29.93**

Table 4.6 Density Values Corresponding to Battles of Arab–Israel War of 1973.

Again, the Wilcoxon Signed Ranks Test was applied to the Arab-Israel War with the same model (4.4) and the parameters (4.3) mentioned above. The results of the test are:

Wilcoxon signed-rank test

data: ArabIsDensity$DensityA and ArabIsDensity$DensityD

signed-rank normal statistic with correction Z = 1.6217, p-value = 0.0524 (4.10)

alternative hypothesis: true mu is greater than 0

As the resulting p-value (4.10) shows, the $\alpha = 0.05$ is slightly less than the p-value of 0.0524. Although the p-value of the model is bigger than the alpha level of 0.05, the difference is not enough to say whether the null hypothesis of the model should be rejected. Actually, the average density of the attacker, 35.03, is slightly greater than the average density value of the defender, 29.93. Thus, to accept the campaign as a tie condition regarding density values of the attackers and the defenders is valid. This result, however, does not change the course of the overall analysis.

D. CONCLUSIONS

Greater dispersion of combat troops on the battlefield is the principle reason for a decrease in casualties despite an increase in weapons lethality. This greater dispersion has occurred in response to increasing lethality of new weapons. As lethality increased, tactics, such as increasing the dispersion of combat forces, were adopted to minimize the effectiveness of the enemy's weapons.

The way in which the increase of dispersion has occurred is shown in Table 4.7. Also, the relation between the density and the dispersion grabs an attention: the decrease in the density means an increase in the dispersion [Table 4.7]. Figure 4.1 and Figure 4.2 give a clear picture of the trend through history concerning the dispersion and the density. It is well obvious that the dispersion increases through history as it is expected to be a counter measure against the increase of the lethality in weapons.

Density and dispersion of troops through history (man per km^2)						
Year	Name of the battle	Density of the attacker (man/km^2)	Density of the defender (man/km^2)	Dispersion of the attacker (m^2/man)	Dispersion of the defender (m^2/man)
1805	Napoleonic War	4494.420	3232.000	222.498	309.406
1861	American Civil War	3378.293	2838.674	296.007	352.277
1870	Franco-Russian War	2125.886	1411.707	470.392	708.362
1914	WW - I	345.168	186.745	2897.142	5354.895
1944	WW - II	72.584	30.902	13777.154	32360.108
1973	Arab-Israel 1973	35.031	29.928	28546.274	33413.389

Table 4.7. Density and Dispersion Values of Campaigns. Note that unit area for density is km^2 while it is m^2 for dispersion.

Comparing Figure 4.1 of the density and Figure 4.2 of the dispersion is important. Even though the attackers' density values are always higher than those of the defenders' in Figure 4.1, all the attackers' dispersion values are below those of the defenders' in Figure 4.2. Mainly, this exists for two reasons.

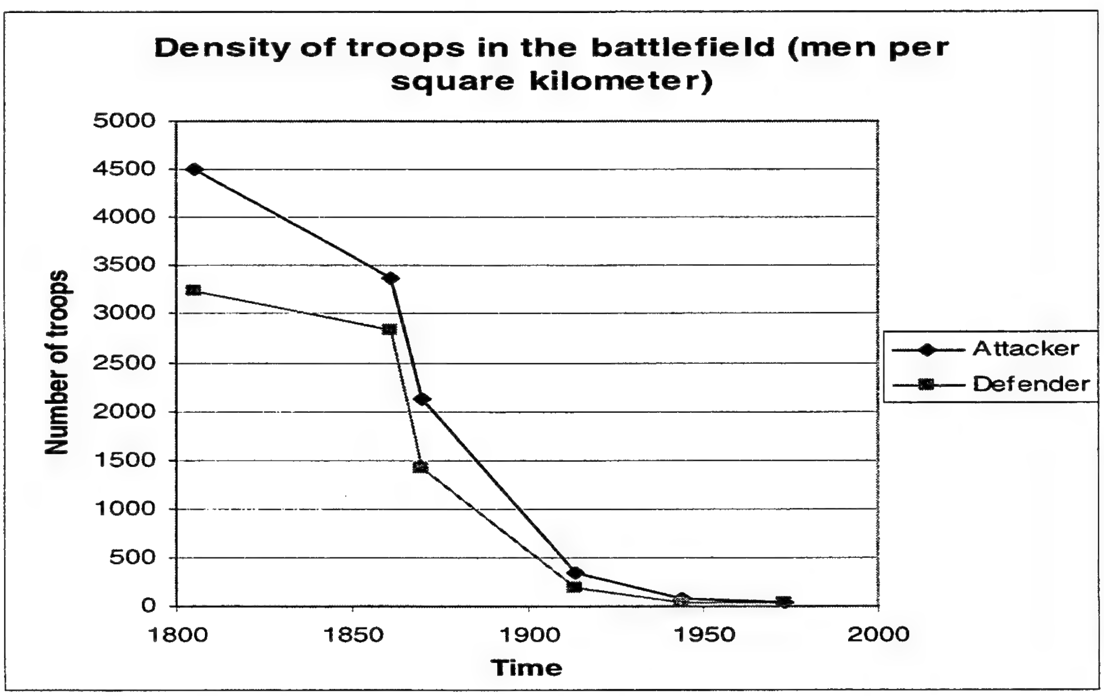

Figure 4.1. The Densities of the Campaigns. Note that the unit area is km^2.

The first reason is that an inverse relation between the density and the dispersion exists. As a result, the smaller the value of the density for a campaign: the bigger the value of the dispersion, which is the inverse of the density. The second reason is that the attacker tends to disperse its forces less than the defender so that the attacker has the opportunity to concentrate its forces against the weaker elements of the defense.

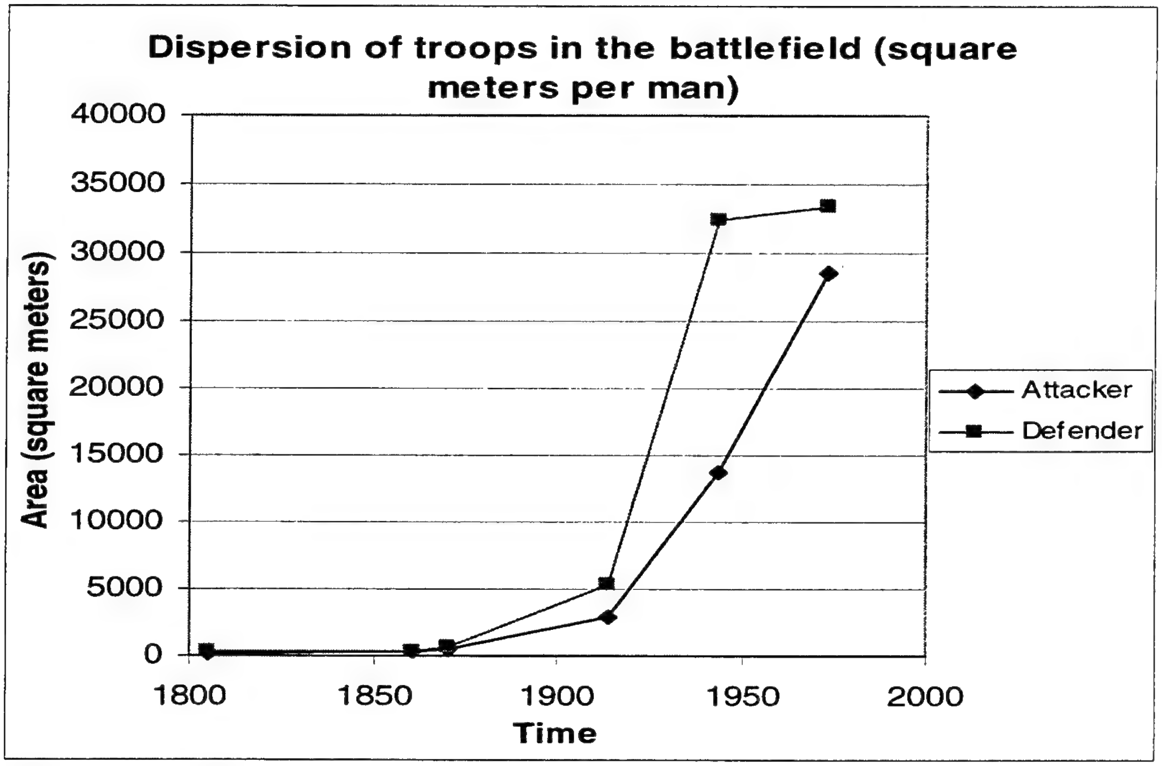

Figure 4.2 The Dispersions of the Campaigns. Note that the unit area is m^2.

Also, it is important to test the hypothesis that density of troops decreases as dispersion of the troops increases in history. To test easily, one must figure out if density and dispersion varies with respect to a change in time. The Kendall's Tau (τ) seems to be the best way of evaluating the result. Also, hypothesis testing on density and dispersion must be conducted for the attacker and the defender, so that there are four hypothesis testings. The model for the hypothesis testing for density is:

Ho: Density and Time are independent.

Ha: A decrease in density is associated with an increase in time. (4.11)

Also, a similar model for dispersion can be used as:

Ho: Dispersion and Time are independent.

Ha: An increase in dispersion is associated with an increase in time. (4.12)

When the Kendall's Tau (τ) is calculated for density values, the results for the attacker and the defender are respectively:

Kendall's rank correlation tau

normal-z = -2.818, p-value = 0.0048

alternative hypothesis: true tau is not equal to 0

sample estimates:

tau

-1 (4.13)

Kendall's rank correlation tau

normal-z = -2.818, p-value = 0.0048

alternative hypothesis: true tau is not equal to 0

sample estimates:

tau

-1 (4.14)

In both cases, the p-value of 0.0048 is far less than the alpha level of 0.05 meaning that the null hypothesis is rejected in favor of the alternative hypothesis. Therefore it is concluded that density for both the attacker and the defender decreases as the time increases. As a result, the sooner the time, the smaller the density. Also, the Kendall's Tau of –1 (4.14) indicates a perfect negative relationship between time and density. One of them increases as the other one decreases.

The Kendall's Tau (τ) is also calculated for the dispersion values of the attacker and the defender:

normal-z = 2.818, p-value = 0.0048

alternative hypothesis: true tau is not equal to 0

sample estimates:

tau

1 (4.13)

normal-z = 2.818, p-value = 0.0048

alternative hypothesis: true tau is not equal to 0

sample estimates:

tau

1 (4.16)

The same p-value of 0.0048 (4.15) causes the rejection of the null hypothesis in favor of the alternative stating that dispersion increases as the time increases. The Kendall's tau value (4.16) of 1, on the other hand, indicates a perfect positive relationship between dispersion and time. It means dispersion increases as the density increases.

In general, there must be a limit to how far armies can be dispersed before they either lose the ability to act in a coordinated fashion or overflow the boundary of the battlefield. This topic itself is both relevant and expansive enough to be a subject for further researches.

V. DAILY CASUALTY RATE

A. INTRODUCTION

This chapter examines the concept of casualty rate, specifically the daily casualty rate (DCR). Here, casualties refer to those killed in battle. Among many interactions relating to casualty, three factors are analyzed:

1. Historical trends in the DCR,
2. The size of the unit vs. the DCR,
3. The battles of each campaign in time sequence vs. the DCR.

The figures and the tables in this Chapter are based on the CDB90FT data set, which is the compilation of 660 major battles. There will be three different analyses, each corresponding to a single factor of interest listed above. The average DCR's of the attacker and the defender are used to determine the historical trends in the casualty rate. After we calculate the DCR for each individual battle, the results are averaged, a number representing the average DCR for either the attacker or the defender in the campaign.

The plotting of casualty rates vs. the unit sizes reveals how the size of the units affects the DCR in the campaign. The compilation of 17 campaigns reveals the historical interaction between the unit size and the DCR. Also, the battles in each campaign are plotted against their corresponding DCR's chronologically so that the nature of each campaign in terms of the DCR is examined. This gives the analyst the opportunity to see how the DCR changed in the duration of the campaign.

B. THE CONCEPT OF CASUALTY RATE

Casualties can be understood best by referring to rates. The actual number of casualties, while important for a single combat event, does not permit aggregation or

comparison among many combat events, because of variations in assigned personnel in different units. Accordingly, calculating casualty rates when performing analyses of casualties is necessary.

With several different kinds of casualty rates, knowing which rate is being used in a particular analysis is critical. Comparing data with different rates can give misleading results.

The three important dimensions of casualty rates are [Ref 5.1]:

1. The duration for which the rate is calculated,
2. The size of the units involved in the battle,
3. The level of combat.

Determining the actual duration of the battle [Ref 5.1] is one of the main factors affecting the calculation of the DCR. A battle is assumed to last between the first day of the conflict and the day the fighting ends. However, the actual duration of the battle used to calculate the DCR is different from simply subtracting two dates. Actually, we must define the duration of a battle to mean only that period when troops are actively engaged in combat. Thus, any lulls in the battle should be ignored so that the actual DCR can be calculated. The actual DCR for a division, for example, is likely to be much higher than the DCR for the same division over an entire campaign consisting of several battles, numerous engagements, and the time spent in reserve.

The size of the unit involved [Ref 5.1] is also crucial. Some scholars strongly claim that the CR is usually inversely proportional to the size of the unit. They argue that small units have higher CR's than large units. The level of combat [Ref 5.1] should be specified for each CR. The difference is due to the proportion of time in which units are

committed to combat at each level. The units in an engagement will be committed to combat during the bulk of that engagement. During a campaign, however, there are periods when the unit is not in combat and has few or no battle casualties. This chapter analyzes 17 campaigns on the basis of battles.

A *rate* [Ref 5.1] is the number of casualties or losses divided by the time period for which the rate is calculated. The most common CR is the DCR. Sometimes, the monthly rate as well as the annual rate is used.

CR's are usually expressed as a proportion of the strength of a unit, which diminishes with each time period. Sometimes the rates are stated as a number of casualties per period. Yet most commonly, the strength that is lost per period is expressed as a percentage. In keeping with this convention, in this chapter, CR's are stated in terms of percentages.

The formula for calculating the DCR is:

$$DCR_A = [d_A / A] / t \tag{5.1}$$

$$DCR_D = [d_D / D] / t \tag{5.2}$$

where DCR_A = the DCR of the attacker,

d_A = the casualty number (those killed in battle) of the attacker,

A = the total strength of the attacker,

DCR_D = the DCR of the defender,

d_D = the casualty number (those killed in battle) of the defender,

D = the total strength of the defender,

t = the active duration of the battle.

C. ANALYSIS OF THE DAILY CASUALTY RATE (DCR)

1. Thirty-Year War—(1620-1648)

The Thirty-Year War is the first campaign for the DCR analysis in the data set. As mentioned before, this campaign is expected to have the highest DCR values for the attacker and the defender. The campaign had 27 days of activity that averaged to 1.5 days per battle. The average strengths of the attacker and the defender [Table 5.1] are 22,694 and 24,367 respectively, which are well below 30,000, except for the battle of Nuremberg in which the defending Imperial Force of 60,000 fought against the Swiss Army of 36,000.

The average DCR values of the attacker and the defender are 0.20 and 0.24 respectively showing that one out of five fielded troops was killed per day in the campaign. Note that the DCR value for the defender is greater than that of the attacker.

Thirty Years War 1620						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.5	22694.22	24367.78	5038.889	6466.667	0.200798	0.242026

Table 5.1. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

The plotting of the size of the unit vs. the DCR values indicates the fact that small units have relatively larger DCR values than those of the large units. However, it is not the ultimate decision for the analysis until the very last campaign is examined [Figure 5.1].

Meanwhile, some of the relatively small units have quite small DCR values, even smaller than those of the large units. The factor implies that the smaller units have bigger

DCR values is that the upper limit of the DCR values decreases as the size of the unit increases.

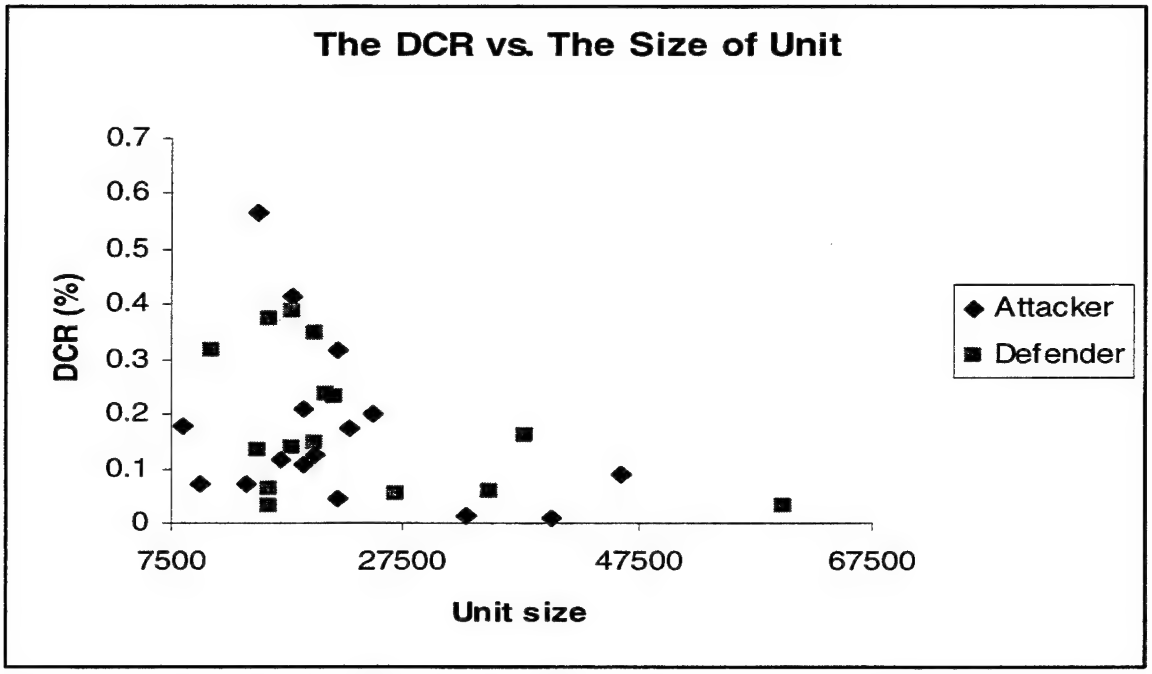

Figure 5.1. Change of the DCR Values with respect to the Unit Size.

The DCR values of the attackers fluctuate during the campaign, but they usually remain below the DCR values of the defenders. After reaching its local maximum point in 1634, the DCR value gradually decreases below 10% for the attackers. In 1645, the highest value of the DCR for the attackers is reached during the battle of Bavaria in which the Imperial Army stormed the Swiss Army of 11,000.

The DCR values for the defenders usually remain above those of the attackers. In the 1636–1643 period of the campaign, the defenders suffered the highest DCR's of 0.6, 0.5, 0.53, and 0.38, which means the defenders lost the corresponding portion of their forces in the battles [Figure 5.2].

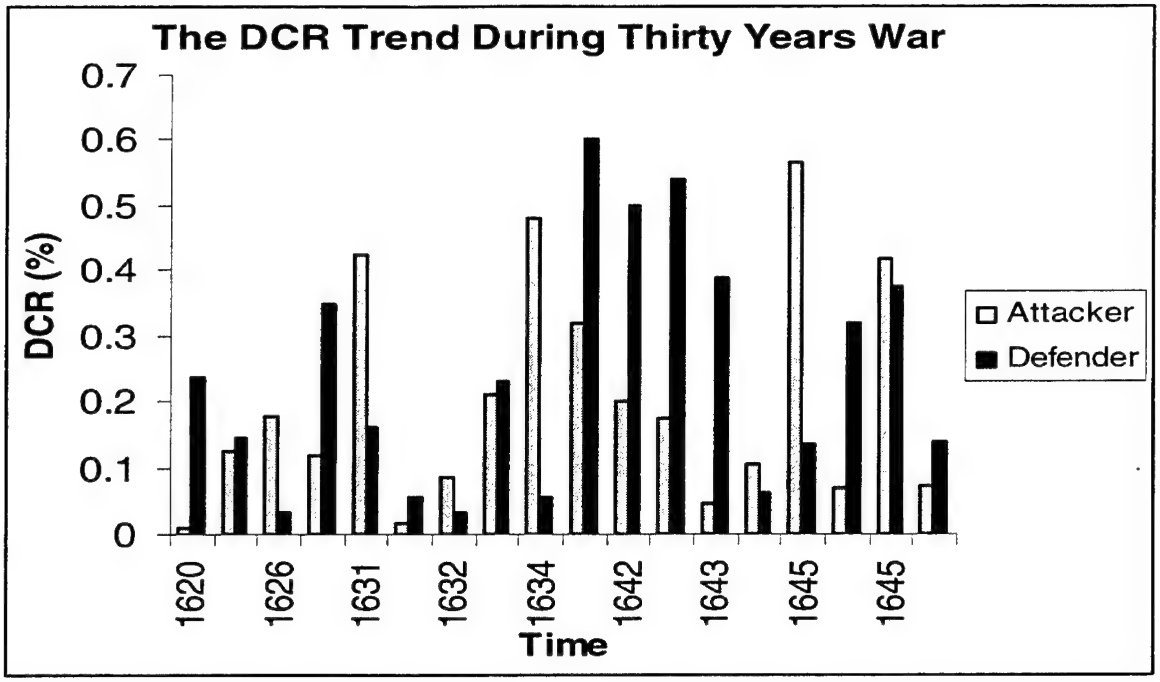

Figure 5.2. The Change in the DCR Values during the Campaign.

2. English Civil War—(1642-1645)

Although the English Civil War is expected to have smaller DCR values than those of the Thirty-Year War's, both average DCR values for the attacker and the defender, 0.22 and 0.29 respectively, are higher than the previous ones. A couple of reasons can be mentioned for these results. First, the number of the battles is small in the campaign, which gives each entry relatively higher power to affect the average result. Any individual fluctuation in a battle has greater impact on the average value. Second, the data concerning the Naseby Battle in 1645 is not available (NA). The probable small DCR values of the battle would decrease the average DCR values. However, another important trend is still preserved: the average DCR value of the attacker is smaller than that of the defender in the campaign.

The battles are fought between forces below 20,000 for the attacker and the defender, 14,000 and 12,434 respectively. The average duration of a battle is 1.16 days. The average DCR values are 0.22 and 0.29 that show the percentage of the losses for the attacker and the defender respectively [Table 5.2].

English Civil War 1642						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.166667	14000	12434	2251	3200	0.225135	0.296359

Table 5.2. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

Despite having a small number of battles, the campaign preserves the trend that large units have smaller DCR values than those of small units. However, this trend is not an absolute result governing all battles. The battle of Marston Moor [Figure 5.3] with defending English Parliamentary Army of 17,500 has a larger DCR value, 0.34, than those of three previous ones.

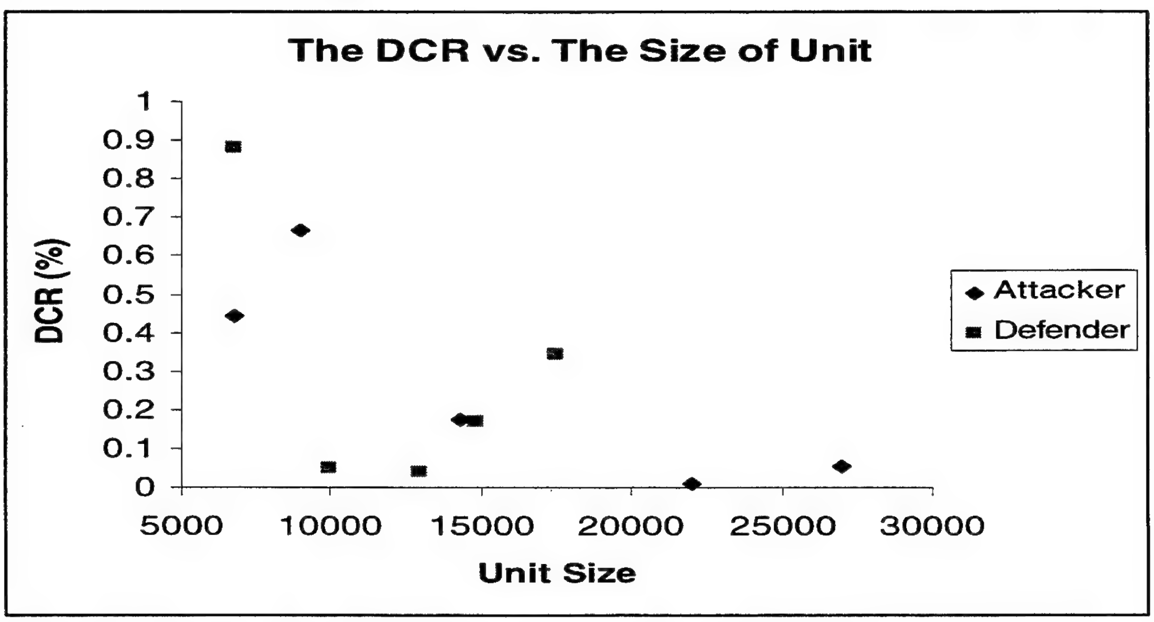

Figure 5.3. Change of the DCR Values with respect to the Unit Size.

Also, four out of six battles in the campaign were fought in 1644. The defender side suffered higher DCR for almost all of the battles except the battle of Naseby in 1645, for which available data concerning the defender's losses does not exist. No intense fighting [Ref 4.1] occurred until the late 1644 when the Scot Army attacked. The defending Scot Royal Army suffered nearly total annihilation of 88% [Figure 5.4].

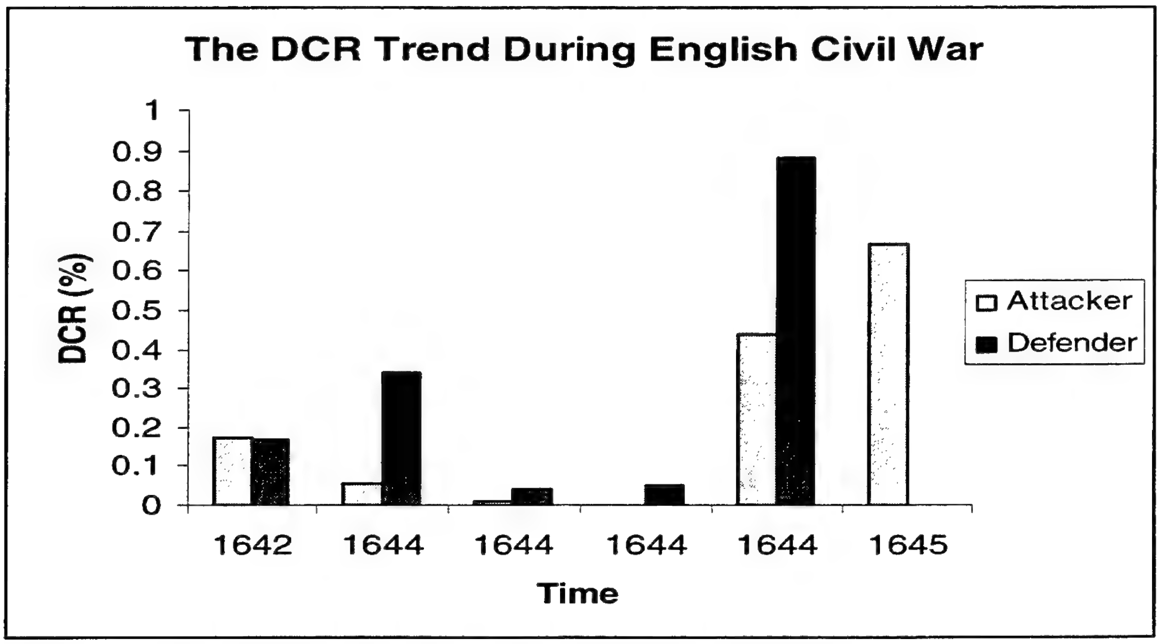

Figure 5.4. The Change in the DCR Values during the Campaign.

3. King William's War—(1689-1693)

The campaign began with the 2,800-strong Scot Army's assault on the 3,400-strong English Army at Killiecrankie in 1689 [Ref 1.7]. The battle was bloody for both the attacker and the defender with the DCR values of 0.11 and 0.25 respectively.

The King William's War can be divided into two stages [Ref 4.1]. The first was the struggle for the control of the British Isles and the English Channel, which lasted until the capitulation of Limerick and the Jacobite forces in Ireland in October 1691. Since bold generalship was important and success in battlefield was far more significant than

protecting the strength of the army, decisive battles were fought to gain control of the country. Thus, the DCR's remained well over the average DCR for the battles that were fought in this stage of the campaign.

The second stage took place from 1691 to the end of the conflict in 1693. Battles during this period were rather static and inflicting relatively low casualties on either the attacker or the defender side. The DCR values for this period of the campaign fall well below the average DCR. The average DCR values, 0.11 and 0.25, preserve the trend of decreasing steeply in the attacker's average DCR value compared to the previous average DCR values of the attacker [Table 5.3].

King William's War 1689						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1	39100	33425	4162.5	7366.25	0.117406	0.251515

Table 5.3. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

The campaign preserves the trend of decreasing DCR values with increasing unit sizes, but roughly. The battle of Feurus stands as an outlier in terms of the trend that is analyzed. Meanwhile, an important feature of the attackers' DCR values should be mentioned. The DCR values of the attackers start with 0.21 and keep decreasing as the unit size increases until one examines the battle of Boyne in 1690 [Figure 5.5]. After the battle of Boyne, the DCR values of the attackers increase, and then, decrease as the unit size increases.

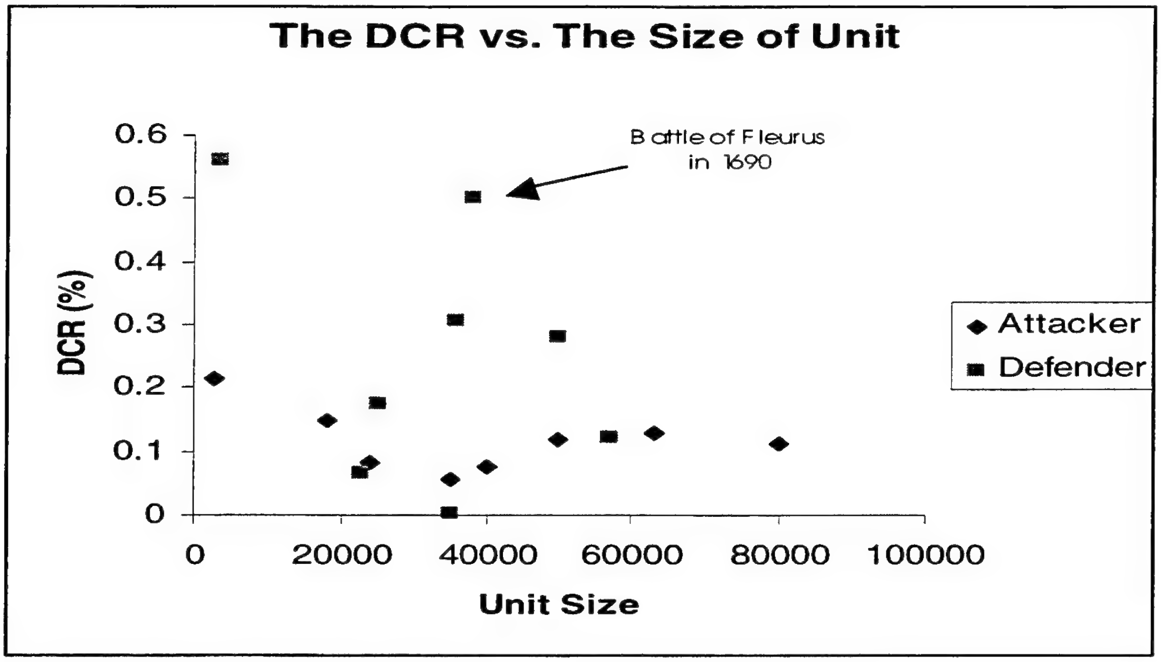

Figure 5.5. Change of the DCR Values with respect to the Unit Size.

During the campaign, the defenders suffered more casualties than the attackers in seven of eight battles. The battle of Walcourt, in which the French army attacked the Allied Army, stands as an example for the case that the attackers suffered more casualties than the defenders, numerically 8% for the attacker and 0.3% for the defender.

In 1689 and 1690, the defenders suffered the highest CR's with 0.55 and 0.5 DCR values [Figure 5.6]. This shows how disastrous the situation was for the defender in those battles since the defenders lost more than half of their total strength in a day. After 1691, the DCR values for the attackers gradually decrease, as DCR values of the defenders fluctuate somewhat. In this year the second stage of the campaign began in which the battles were static rather than decisive.

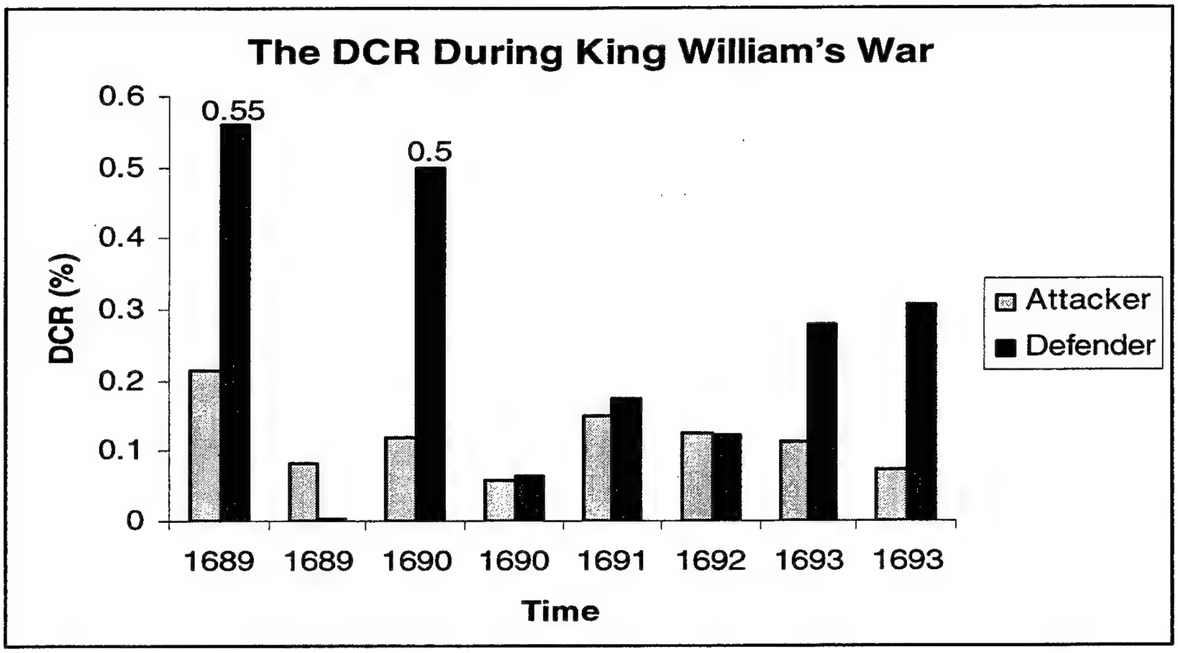

Figure 5.6. The Change in the DCR Values during the Campaign.

4. Austrian Succession War—(1741-1745)

The Austrian Succession War was severely affected by political considerations and by tactical changes in the battlefield, although these did not make the battle any less hard fought. The average DCR values maintain the trend that the defenders have higher DCR's than the attackers: 0.17 for the attackers and 0.18 for the defenders.

However, this campaign is one of two periods in history that the average DCR values for the attackers have an increasing trend. Special attention should be paid to the point that the difference between the average DCR for the attackers and the average DCR for the defenders becomes smaller than previous ones: less than 1%.

The average strengths of the attackers and the defenders are well over 30,000: 33,666 for the attackers and 38,671 for the defenders. Also, the average strength of the attackers, which is expected to be greater than of the defenders, is actually less [Table 5.4].

Austrian Succession War 1741						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1	33666	38671.43	5570.857	6702.714	0.172719	0.18067

Table 5.4. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

The Austrian Succession War is rather unique, for it serves as an example that greatly contradicts the trend that the DCR values decrease as the unit size increases. Actually, this war does not support any particular trend. The DCR values fluctuate as the size of the unit increases and gives no indication that the DCR values interact with the size of the unit [Figure 5.7]. This fluctuation may be due to new tactics that were applied on the battlefields in The Austrian Succession War. The nature of the conduct in the campaign [Ref 4.1] has some factors, such as multinational armies, and drastic changes in the tactics that might have contributed to variations in the campaign. However, determining the reason behind this is complex.

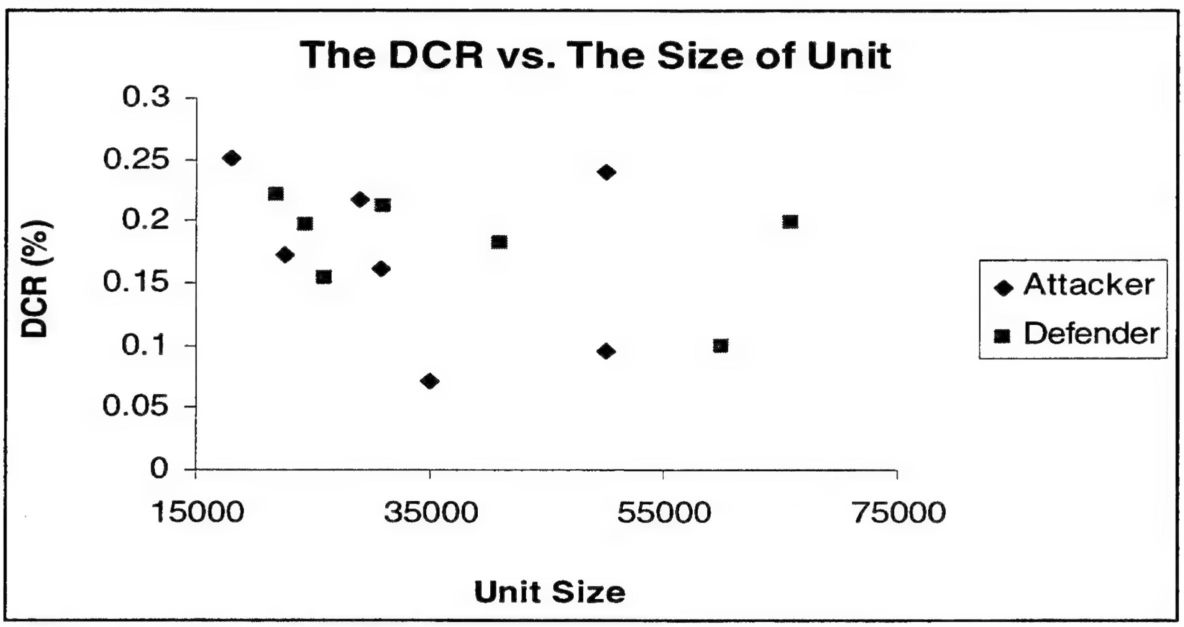

Figure 5.7. Change of the DCR Values with respect to the Unit Size.

When the duration of the campaign and the DCR values of the battles are examined, the DCR values of either the attacker or the defender obviously remain below the level of 0.3, which is assumed to be the annihilation rate for any army suffering that much casualty. Possibly, the campaign was conducted by the generals with respect to 30% CR [Figure 5.8]. If this is the case, the thumb rule of 30% CR can be extended to the era of the Austrian Succession War. The CR values during the campaign remain between 10% and 27% with an exception of the Dettingen Battle in which attacking British forces suffered less than 5% casualty.

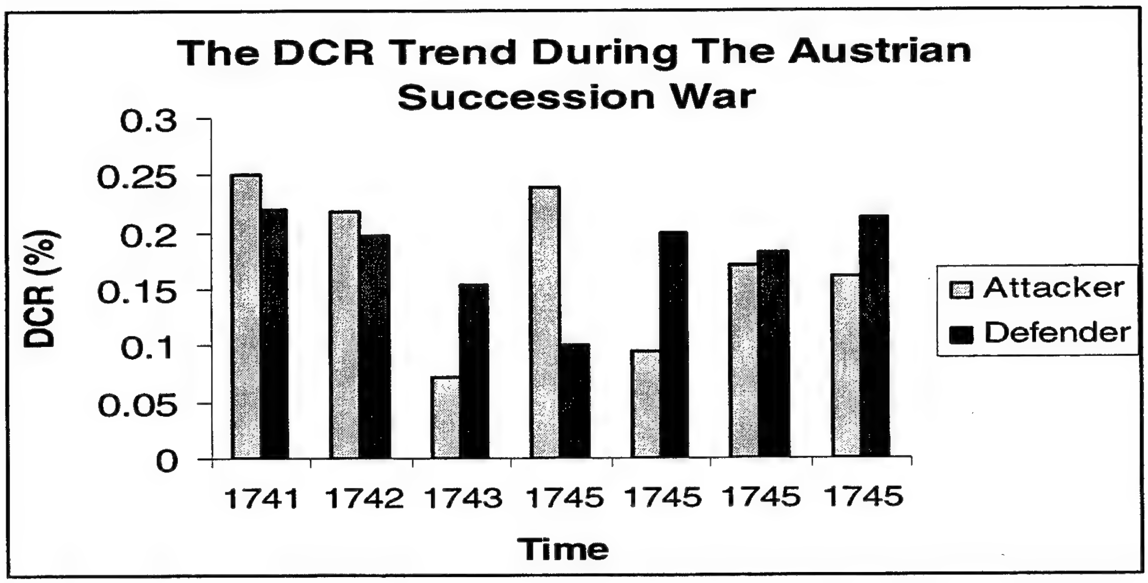

Figure 5.8. The Change in the DCR Values during the Campaign.

5. The Seven Years War—(1756-1760)

The Seven Years War is in the same group of the campaigns in which the average DCR values for the attackers have an increasing nature. Also, in the Seven Years War the average DCR value of the attacker is greater than that of the defender: 0.18 for the attacker and 0.17 for the defender.

The reason for the change in the DCR values might be the tactics of the firepower. The Prussians responded to Austrian and Russian strength with a number of innovations [Ref 4.1]. They used artillery to open deadlock battlefronts, distributed 12-pounder cannons among the infantry in 1759 and 1760, and employed howitzers and explosive shells for offensive purposes.

The average casualties for the attacking Austrian and Russian forces increased as a result of the artillery-based tactics making the average DCR values for the attackers higher than expected.

Seven Years War 1756						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1	40025	36609.72	7168.278	7456.333	0.189793	0.17192

Table 5.5. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

Comparing the unit size of the campaign with the DCR values does not indicate any trend. The DCR values for battles, for example, vary between 0.002 and 0.45 for the unit size range of 40,000-60,000. Also, the group of battles does not indicate that the DCR values decrease as the unit size increases [Table 5.5].

Artillery [Ref 4.1] proved to be absolutely the dominant factor through The Seven Years War is an absolute reality. Thus, relating irregular DCR values to the innovative use of artillery against old-fashioned tactics of the battlefield is logical. It is well understood that changing the tactics of the battlefield when responding to innovations in technology takes time.

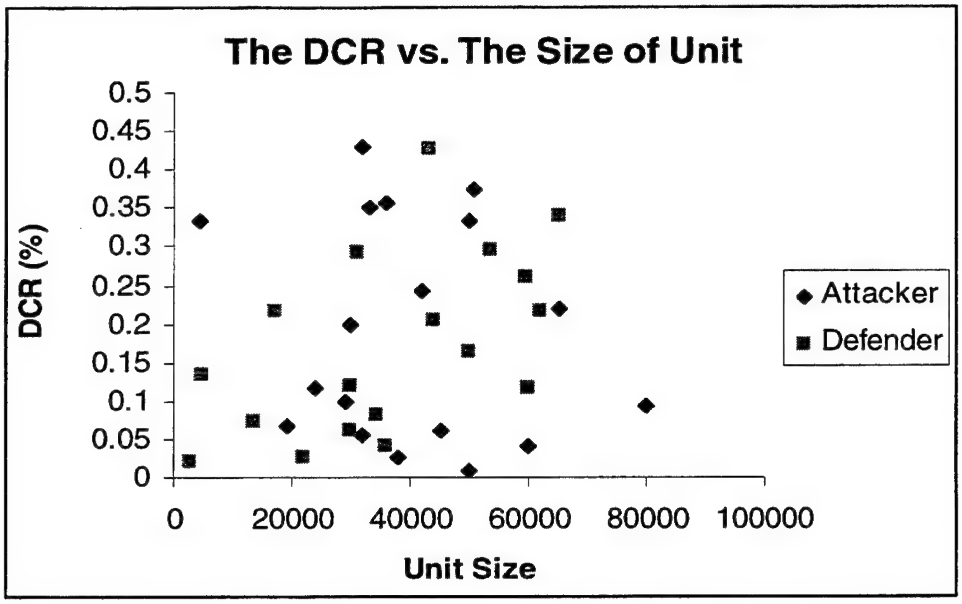

Figure 5.9. Change of the DCR Values with respect to the Unit Size.

The campaign, on the other hand, is full of fluctuations in casualty values. Especially, the period of 1757-1758 has the highest DCR values either for the attackers or the defenders.

The second half of 1757 and 1758 was a period of particular activity [Ref 4.1], with a Russian invasion of East Prussia and victory there at Gross-Jagersdorf on August 30th; a Swedish invasion of Pomerania; the French conquest of Hanover; the raising of the Prussian siege of Prague and the end of the Prussian invasion of Bohemia after the Austrian victory at Kolin; a successful Austrian raid that captured defenseless Berlin on 16 October; and the Austrian capture of most of Silesia on 13 November.

Many decisive battles were fought in that period of the campaign, so the casualties increased for both the attacker and the defender. The attacker suffered the highest DCR of 0.43 in the second half of 1757 while the defender hit the highest value of DCR of 0.42 in 1758 [Figure 5.10].

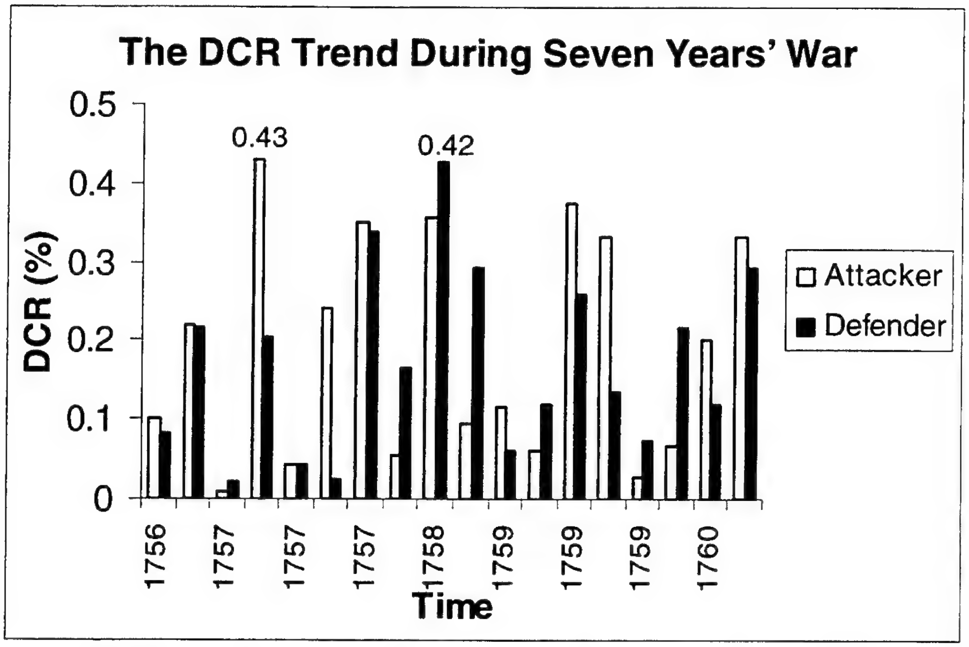

Figure 5.10. The Change in the DCR Values during the Campaign.

6. American Revolutionary War—(1775-1781)

The first major revolutionary conflict [Ref 4.1] ended in the successful overthrow of British authority in the Thirteen Colonies. It was also the first major conflict between regular forces and irregular militia. American militiamen were seeking to defeat a highly trained British army supported by both the largest navy in the world and the strongest system of public finance in Europe.

There were never enough troops in the Continental Army and its size fluctuated greatly causing major problems. The average number of troops in the campaign remained well below 6,000: 4,697 for the attackers and 5,056 for the defenders.

The infantry dominated the battlefield in the colonies [Ref 4.1]. The impact of the trained British musketeers with bayonets that inspired fear among the Americans was lessened by the Americans' ability to entrench themselves in strong positions. Yet attacking American militia suffered heavy casualties causing the average DCR increase

for the attacker: 0.21. Meanwhile, the average DCR value for the defenders decreases, remaining below the average DCR value for the attacker: 0.15 [Table 5.6].

American Revolution 1775						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.071429	4697.857	5056.786	507.8571	419.4286	0.213511	0.150773

Table 5.6. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

Although the campaign has unique features in its conduct, it perfectly preserves the trend that the small units suffer higher DCR values than the large units. Examining the small unit values, one clearly sees that the DCR value corresponding to the level of 1100 troops, for example, varies between 0.07 and 0.84 [Figure 5.11], which is quite a large difference to consider. However, the upper limit of the average DCR values becomes smaller as the unit size number becomes greater.

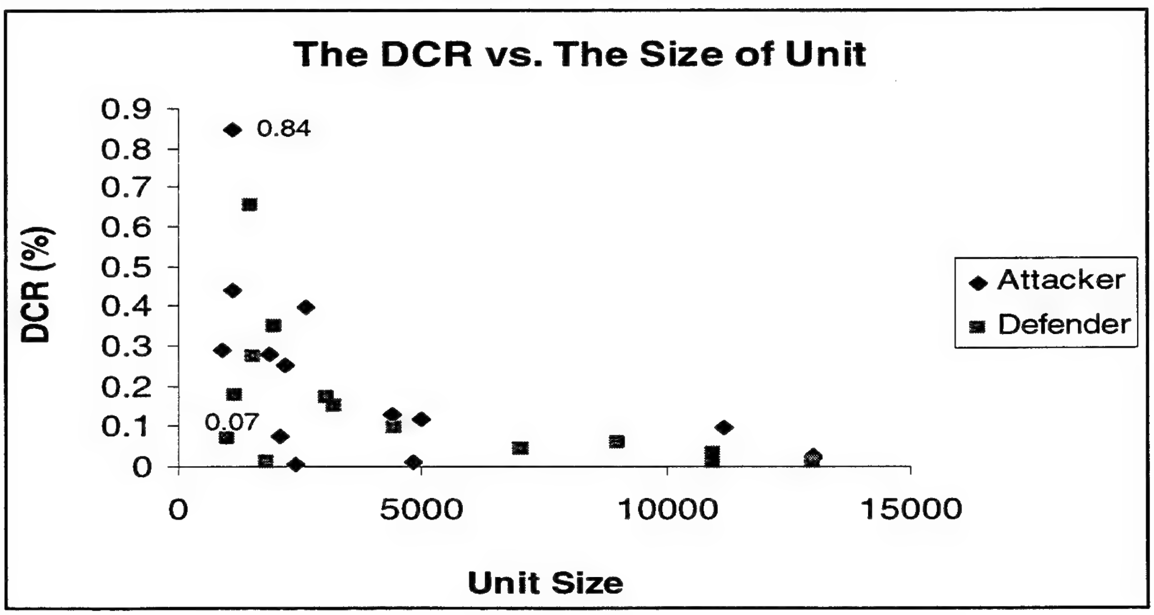

Figure 5.11. The Change of the DCR Values with respect to the Unit Size.

Since the campaign's nature of conduct is quite different from the other campaigns, there are some unexpected fluctuations in the DCR values of both the attacker and the defender. Moreover, claiming that the defender generally suffered more DCR values than the attacker is difficult.

In the battles of Bunker Hill and Guilford, the British army suffered the DCR of 0.65 and 0.84 respectively [Figure 5.12]. These numbers indicate the total annihilation of British armies.

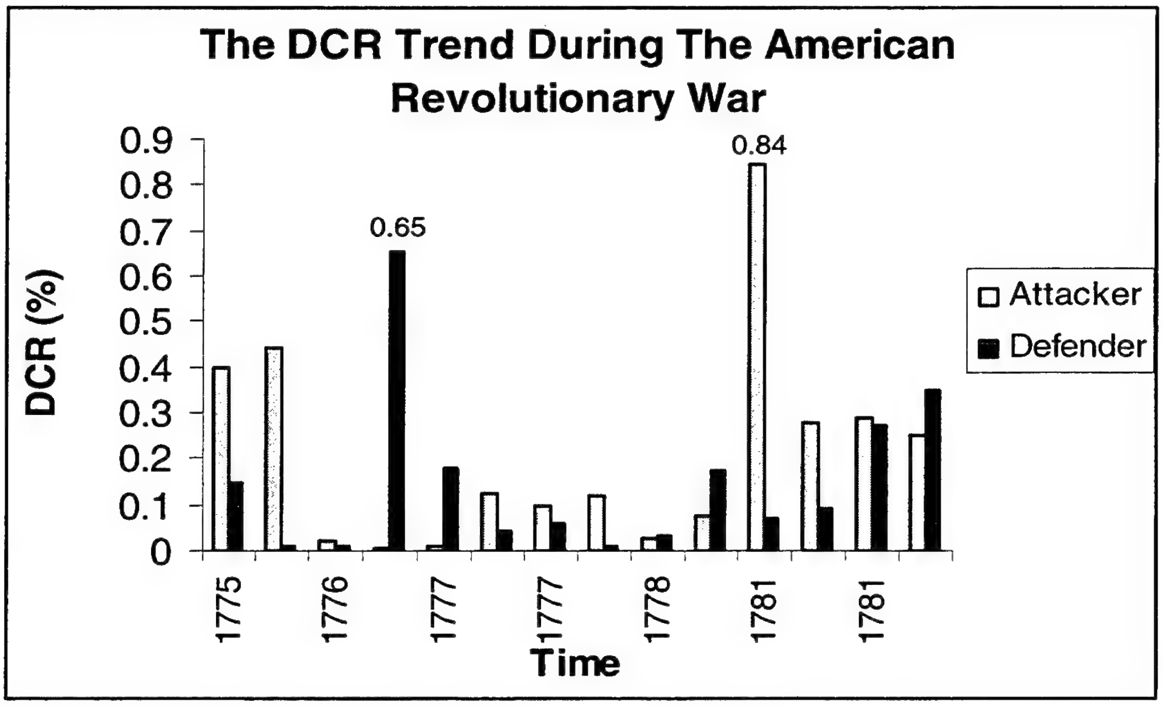

Figure 5.12. Change in the DCR Values during the Campaign.

7. War of the First Coalition—(1792-1799)

In the War of the First Coalition, the average DCR value for the attacker drastically decreased to almost one-third of the previous value of 0.21: 0.08. Meanwhile, the average DCR value for the defender continues to decrease: 0.12, but higher than the attacker's value [Table 5.7].

This campaign has some important aspects. Yet peculiar, *Speed* [Ref 4.1], for example, was a feature of battling and French armies benefited from this fact. Speed in the battlefield is the hidden reason behind the decrease in the DCR value of the attacker. Although the battlefield use of the artillery was of great importance and responsible for inflicting casualties, the attackers responded to the artillery by increasing the mobility of the units, thus, suffering less casualties.

War of First Coalition 1792						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.285714	34807.14	26407.14	3646.429	3343.571	0.080725	0.120988

Table 5.7. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

As a cloud of data points, the unit size comparison of the DCR values do not indicate that the DCR values decrease as the unit size increases. In the battle of Fleurus in 1794, the 73,000 of French army battled the Austrian army. This battle can be treated as an outlier.

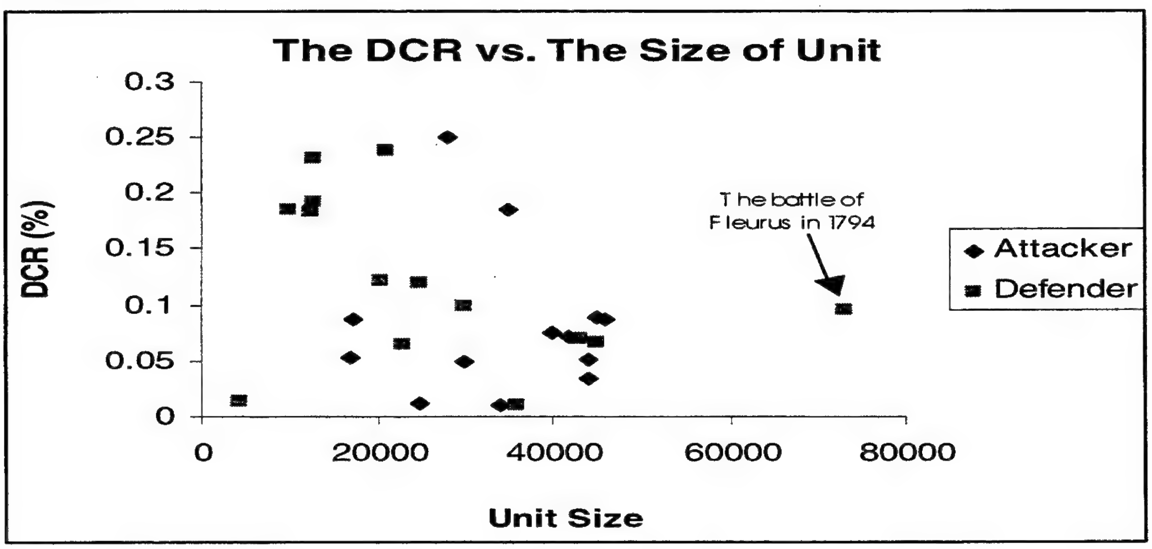

Figure 5.13. Change of the DCR Values with respect to the Unit Size.

One aspect of modern warfare emerges and gains support from the analyses in the campaign that the highest level of CR that an army can assume is 0.3 or less. That aspect appeared first in the Austrian Succession War and was repeated in War of the First Coalition. Obviously, the DCR values of battles remained below 0.25 value for the attacker and the defender: 0.25 and 0.23 respectively [Figure 5.14].

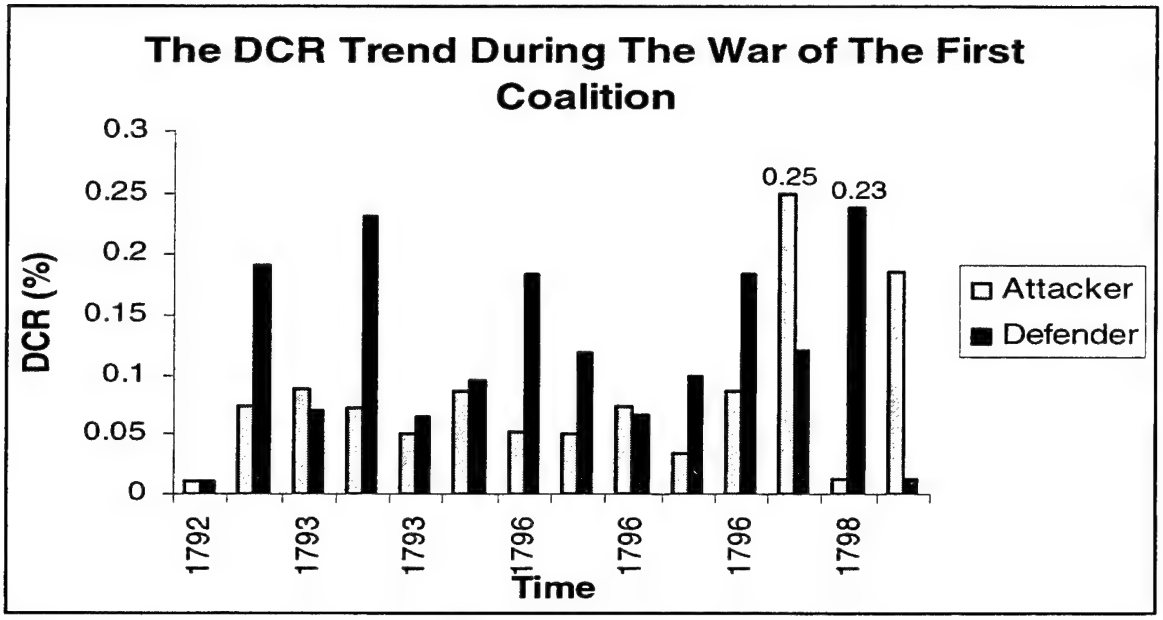

Figure 5.14. The Change in the DCR Values during the Campaign.

8. War of the Second Coalition—1800

The navy's consequential role was assisting Britain to defeat her European rivals in the War of the Second Coalition. Also, the War of the Second Coalition is one of three cases in which the average DCR value of the attacker surpassed the average DCR value of the defender: 0.13 and 0.10 respectively.

This campaign can be evaluated in the same way that the War of the First Coalition was examined because there was no time gap between the two campaigns [Ref 4.1]. Simply, the Second Coalition is another form of the First Coalition having extra combatants in the conflict.

However, one important feature rises from the increase of the average DCR value of the attacker. In this campaign the average DCR value of the attacker gained a positive slope making the value greater than that of the War of the First Coalition. The average DCR value of the defender, on the other hand, continues to decrease as usual.

War of Second Coalition 1800						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.428571	44428.57	39571.43	7985.714	5871.429	0.135075	0.109219

Table 5.8. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

Meanwhile, the unit-wise comparison of the DCR values roughly preserves the trend that the DCR values decrease as the unit size increases. The battle of Hohenlinden in 1800, in which the Austrian army attacked the French positions, remains as an outlier with the DCR value of 0.35 [Figure 5.16]. Excluding the battle of Hohenlinden, one observes that the DCR values become smaller as the unit size becomes large [Table 5.8].

Another point to mention about this campaign is that the unit size of the defenders and the attackers remained well above 20,000. Actually, the average unit sizes of the campaigns should be considered when making inter-campaign comparisons. The average unit sizes vary with each campaign. Thus, comparing the campaigns with relatively close average unit sizes is more accurate.

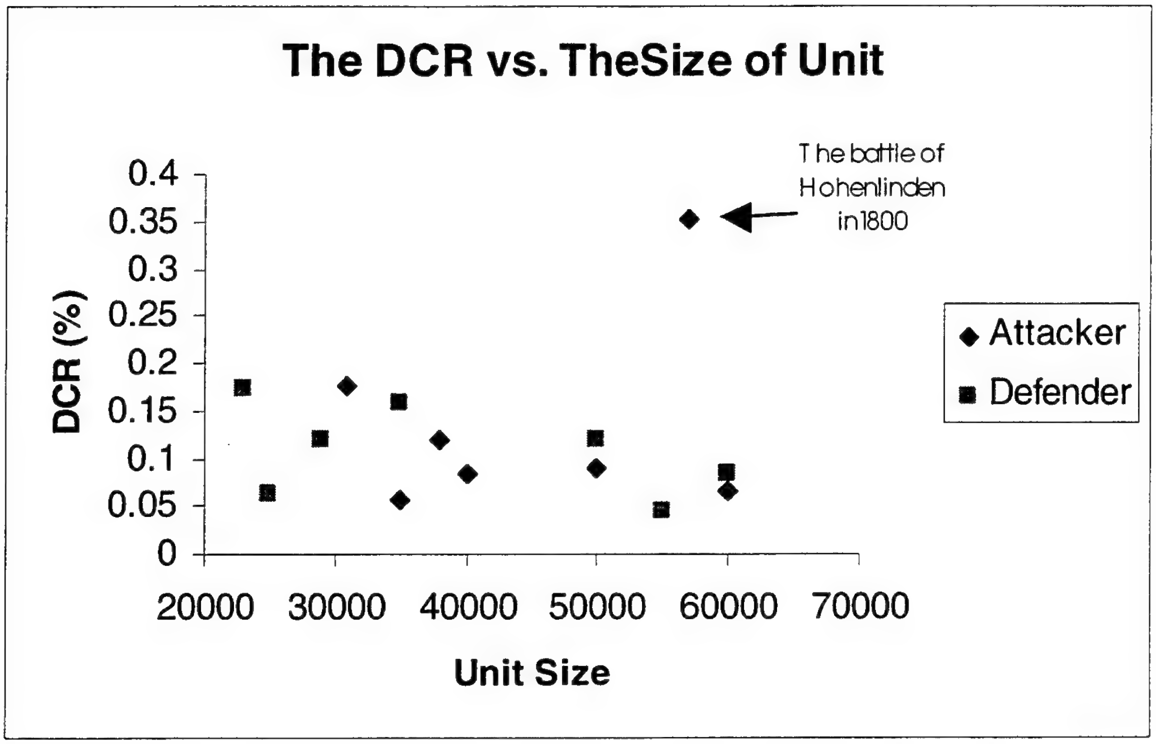

Figure 5.15. Change of the DCR Values with respect to the Unit Size.

The battle of Hohenlinden in 1800 stands as the highest DCR value for the attacking Austrian army: 0.35. The reasons behind this situation might be the high mobility of the defending French army and its effective use of the artillery in the battle.

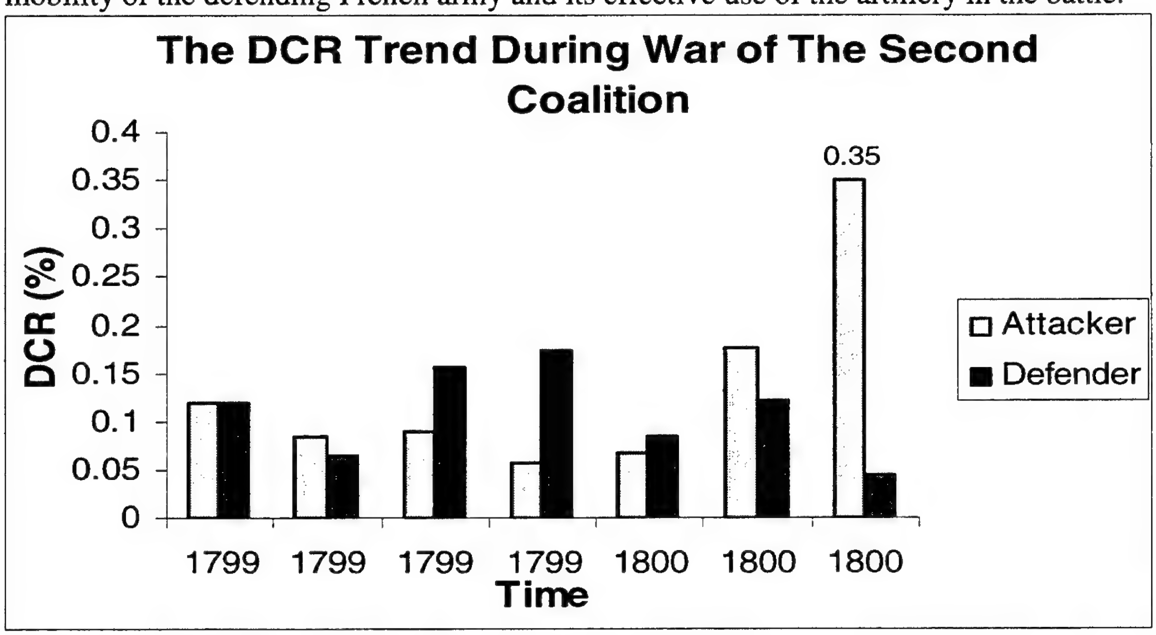

Figure 5.16. The Change in the DCR Values during the Campaign.

9. Napoleonic Wars—(1805-1815)

The Napoleonic War is also one of three cases in which the average DCR values have an increasing nature. Despite being less than the average DCR value of the defender, the average DCR value of the attacker has a slight increase: 0.136 for the attacker and 0.162 for the defender [Table 5.9].

Among the many reasons for the increase in the average DCR values only two seem to have greater importance. The first is the willingness of the commanders to accept a high rate of casualties and to mount costly frontal attacks on fortified positions. General Suvarov of Russia [Ref 4.1], for example, relied on bayonet attacks rather than the use of defensive firepower.

The second is the effective use of the artillery in the battlefield [Ref 4.1]. The generals of the era firmly believed in the efficacy of artillery, especially of 12-pounders, organized into powerful batteries. At Wagram, Napoleon reorganized his attack with a battery of 102 guns.

Napoleonic Wars 1805						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.448276	86205	69751.76	14674.07	15112.45	0.136216	0.16224

Table 5.9. Average DCR Values, Strengths, Casualty Numbers, and Duration of a Battle.

The campaign as a whole is dominated by battles between relatively large armies having an average number of 86205 for the attacker and 69751 for the defender. Battles between relatively large units roughly preserve the trend that the DCR values decrease as the unit size increases [Table 5.9].

Three battles of the campaign, namely Jena in 1806, Borodino in 1812 [Figure 5.17], and Leipzig in 1813, serve as a base to track the relation between the unit size and the DCR value clearly. As the unit sizes of these battles increase, their DCR values decrease accordingly.

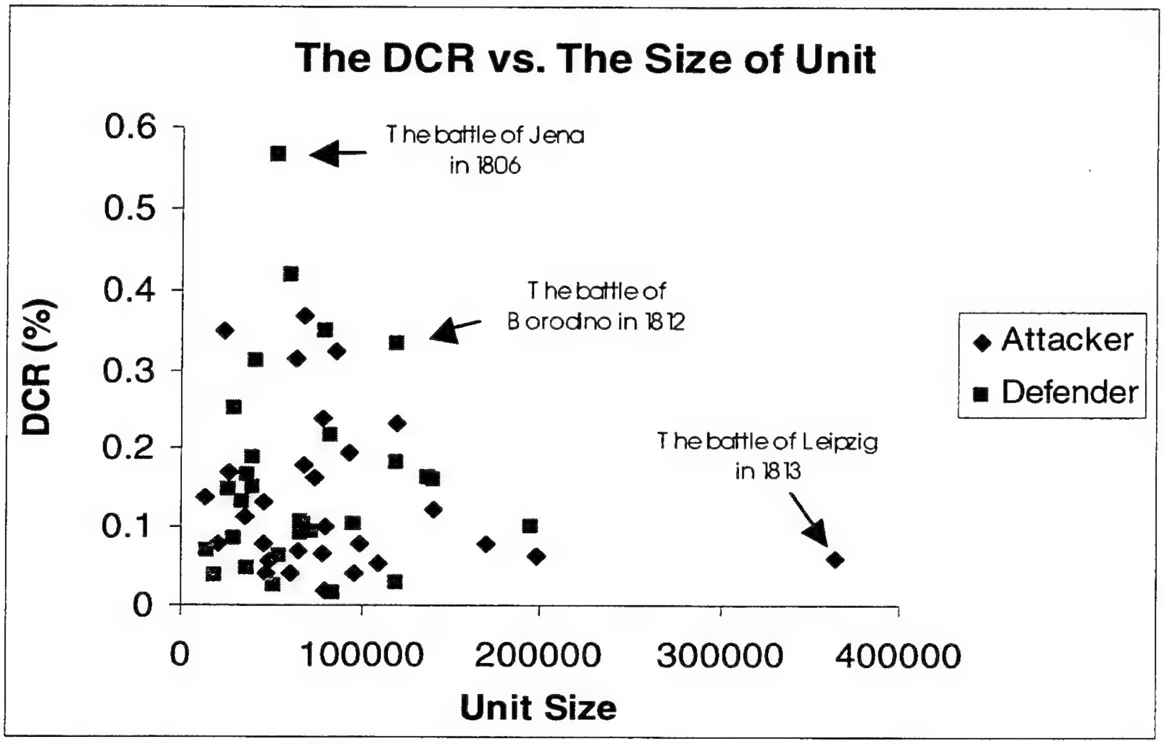

Figure 5.17. Change of the DCR Values with respect to the Unit Size.

If the DCR values during the campaign are examined closely, the most active period of the campaign clearly took place between 1805 and 1807 and in 1812. At Jena in 1806, the attacking French army annihilated 56% of the Prussian army per day. At Friedland in 1807, the same French army attacked the Russian forces and destroyed 41% of its strength.

During 1812, the battles were fought under heavy CR values [Figure 5.18] since both the attackers and the defenders were engaged in decisive battles. The DCR values

for the battles in this period of the campaign remained higher than the rest of the campaign.

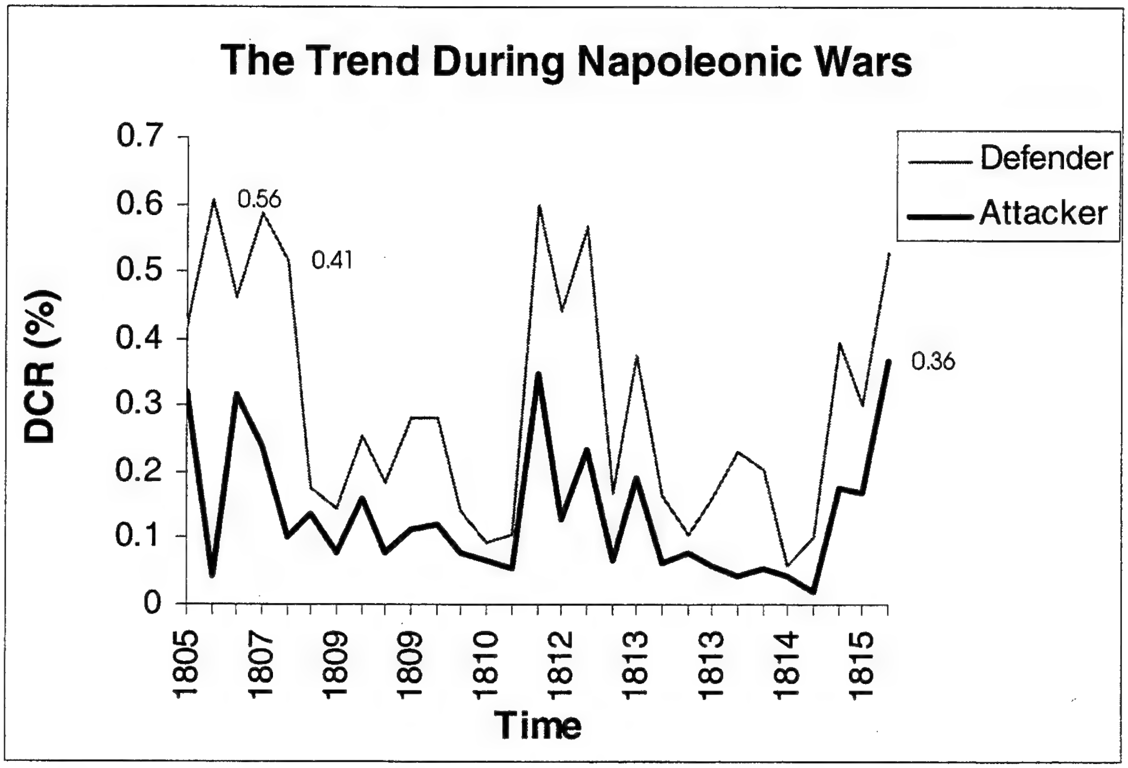

Figure 5.18. The Change in the DCR Values during the Campaign.

10. U.S.–Mexico War—(1846-1847)

The US–Mexico War involved small conflicts in terms of the unit sizes of the opposing forces when compared the Napoleonic War, which involved tens of thousands of troops. Also the average DCR values for the attacker and the defender in the US-Mexican War revealed a decreasing nature, except for an increasing slope during the American Civil War.

The average DCR value for the attacker remains well below the DCR value of the defender: 0.07 and 0.16 respectively. Also, the DCR values in the battles are less than

0.25 with an exception of the battle of Contreras in which the defending Mexican forces suffered a heavy loss of 37%.

US - Mexican War 1846						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.375	6220.625	8732.375	664	1860.375	0.079643	0.161358

Table 5.10. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The comparison of the DCR values regarding unit sizes does not have any trend that the DCR values decrease as the unit size increases. On the contrary, it reflects an increasing nature of DCR values as the unit size increases, excluding the battle of Contreras and the battle of Molino Del Rey [Table 5.10].

On the other hand, when battles in the campaign were fought among relatively small armies with high mobility, the outcome of the casualty fluctuates.

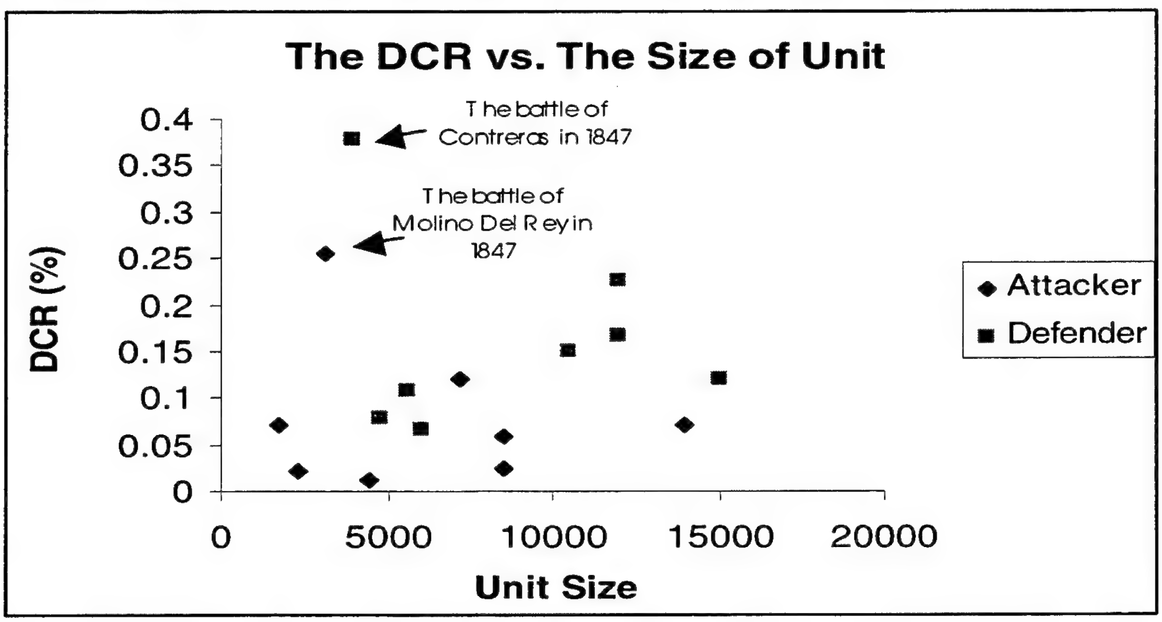

Figure 5.19. Change of the DCR Values with respect to the Unit Size.

The decisive battles of the campaign were all fought in 1847 making the DCR values increase for the attacker and the defender. In the battle of Contreras in 1847 [Figure 5.19], for example, the defending Mexican army suffered a 37% DCR when a slightly superior U.S. army attacked its positions.

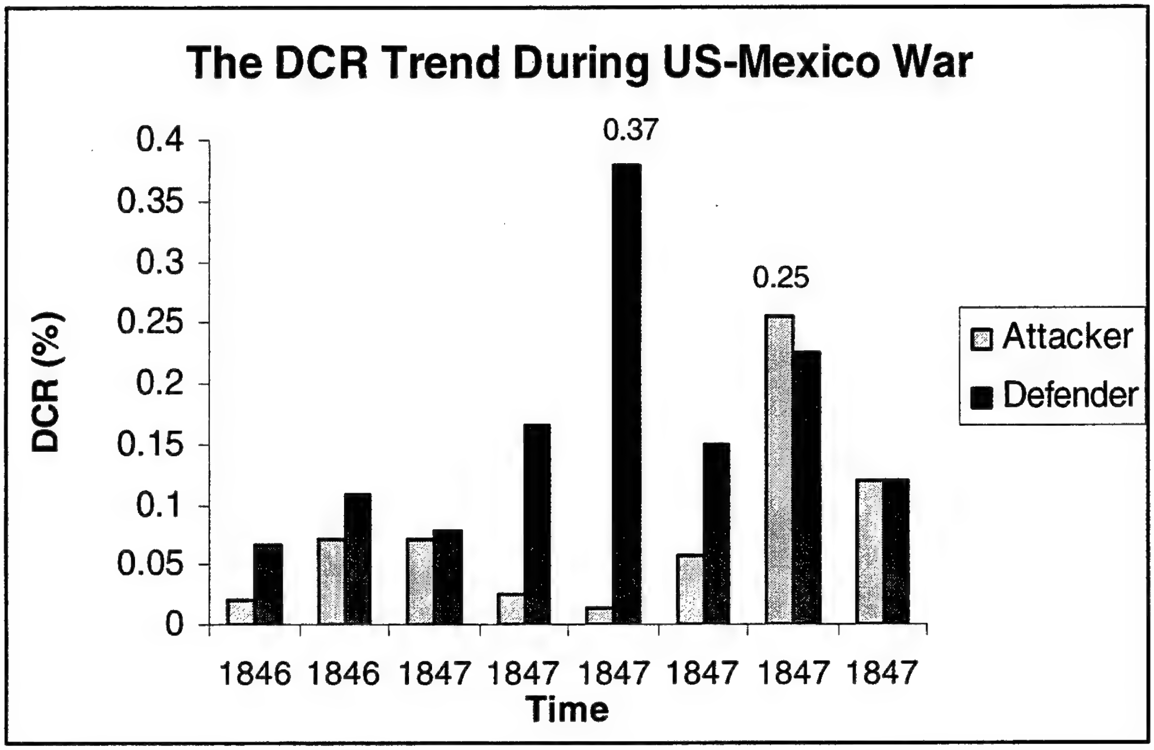

Figure 5.20. The Change in the DCR Values during the Campaign.

11. American Civil War—(1861-1865)

The American Civil War is also one of the three periods in which the average DCR value for the attacker is less than that of the defender. The main reason for the increase in the average DCR value for the attacker and the defender was the introduction of the muzzle-loading rifle musket or conoidal bullet.

The rifle musket [Ref 3.3] was the standard weapon used by both the North and South in the Civil War. The conoidal bullet was lethal at longer ranges than canister or spherical case shot fired from contemporary smoothbore cannon, and could reach almost

as far as solid shot and shell from cannon. Thus, both sides suffered increasing average DCR values during the campaign: 0.08 for the attacker and 0.13 for the defender.

American Civil War 1861						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.979592	40176.33	30663.9	5765.776	4682.837	0.08949	0.134788

Table 5.11. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The unit size played an important role in the DCR values of the battles: the smaller the unit size, the bigger the DCR value. However, the battle of Front Royal in 1862 [Figure 5.21] was an exception in terms of the DCR values. In this battle, the Southern army of 16,000, having a force ratio superiority more than 10-to-1, attacked the U.S. army of 1,063.

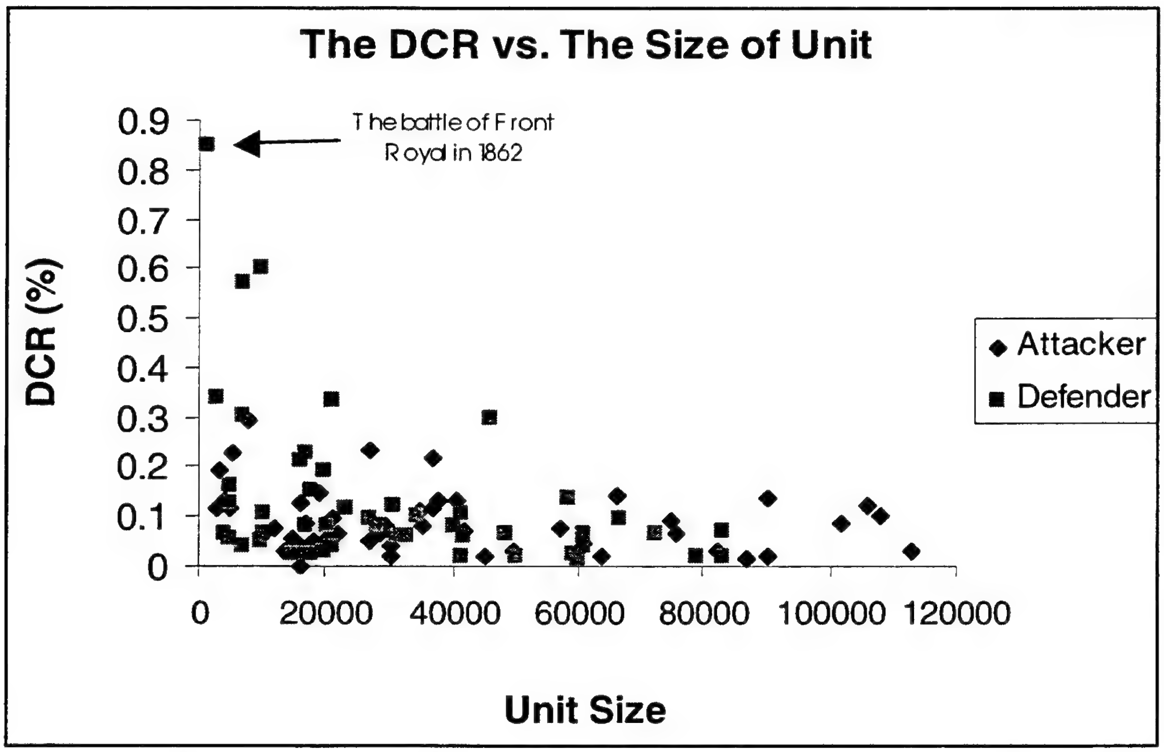

Figure 5.21. Change of the DCR Values with respect to the Unit Size.

The DCR values for the battles remain relatively high in 1862 when the campaign's decisive battles were fought. In the battle of Front Royal [Figure 5.22] in 1862, the U.S. army of 1,063 was almost totally annihilated, 85%, by the numerically superior Southern force of 16,000.

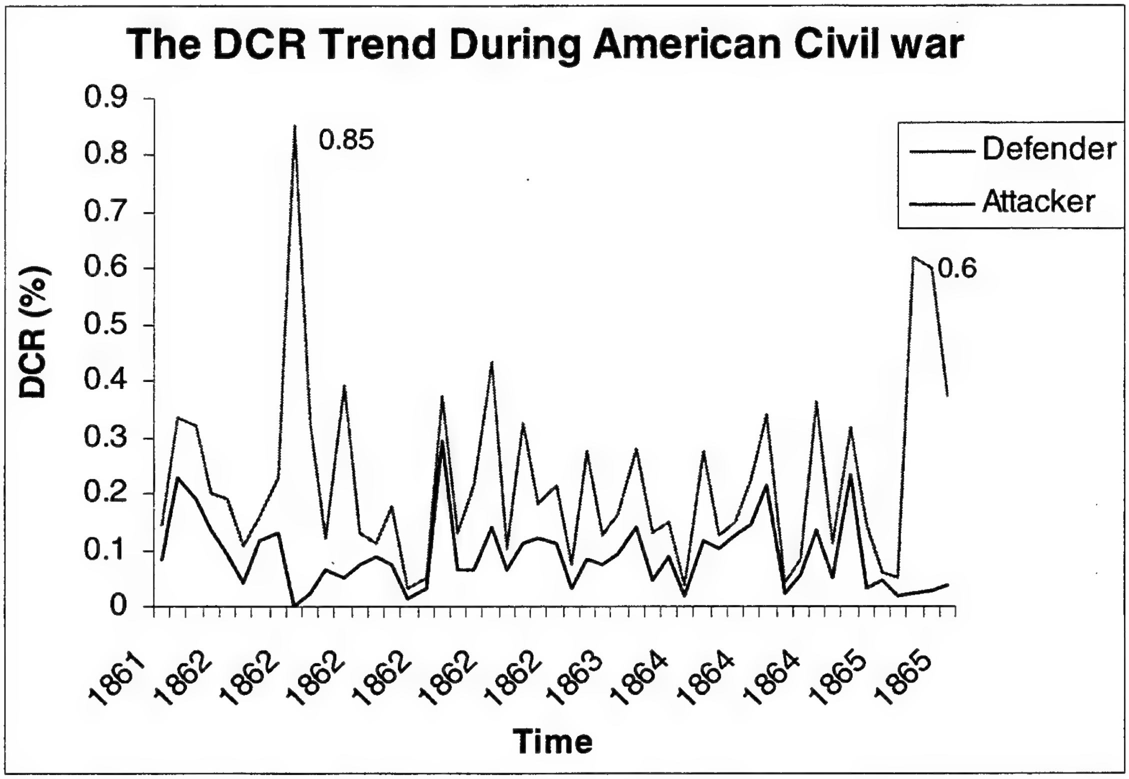

Figure 5.22. The Change in the DCR Values during the Campaign.

12. Franco–Prussian War—(1870-1871)

In the Franco–Russian War, the average DCR value of the defender has a positive slope while the average DCR value of the attacker decreases. The average unit sizes of both the attackers and the defenders are relatively large: 98,100 and 68,500 respectively [Table 5.12].

One of the main reasons behind the increasing DCR value of the defender is that the highly disciplined and trained Prussian army was virtually always in an attacking

posture during the campaign inflicting constant casualties to the defenders. The average DCR value of the attackers remains below the average DCR value of the defender: 0.07 and 0.18 respectively [Table 5.12].

Franco - Prussian War 1870						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
1.5	98100	68500	8250	14810	0.071381	0.184452

Table 5.12. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The relation between the unit size and the DCR values is roughly perceptible when the battles with the highest DCR values are examined. The battle of Sedan has a different feature to consider. The attacking German army of 200,000 troops suffered less DCR value than the defending French army of 120,000: 0.045 and 0.31 respectively.

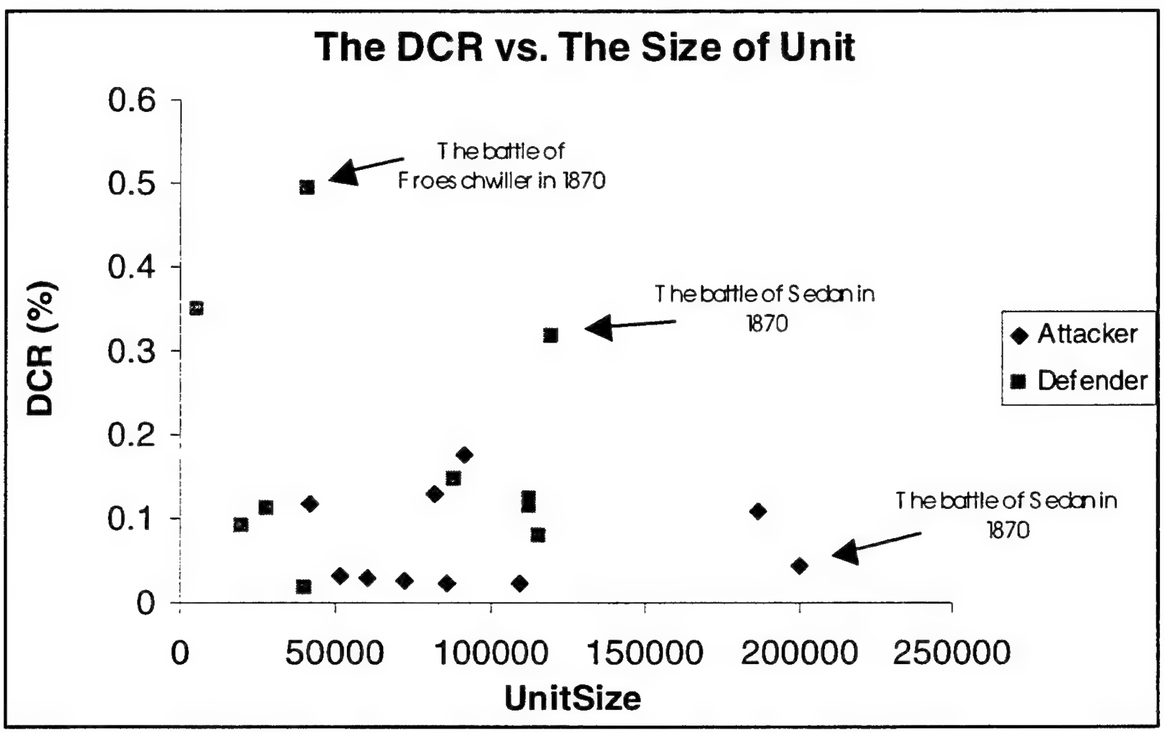

Figure 5.23. Change of the DCR Values with respect to the Unit Size.

A quite interesting feature appears in the DCR values of the attacker during the campaign. While the DCR values of the defender fluctuate between 0.49 and 0.01, the DCR values of the attacker follow a pattern obvious in Figure 5.24.

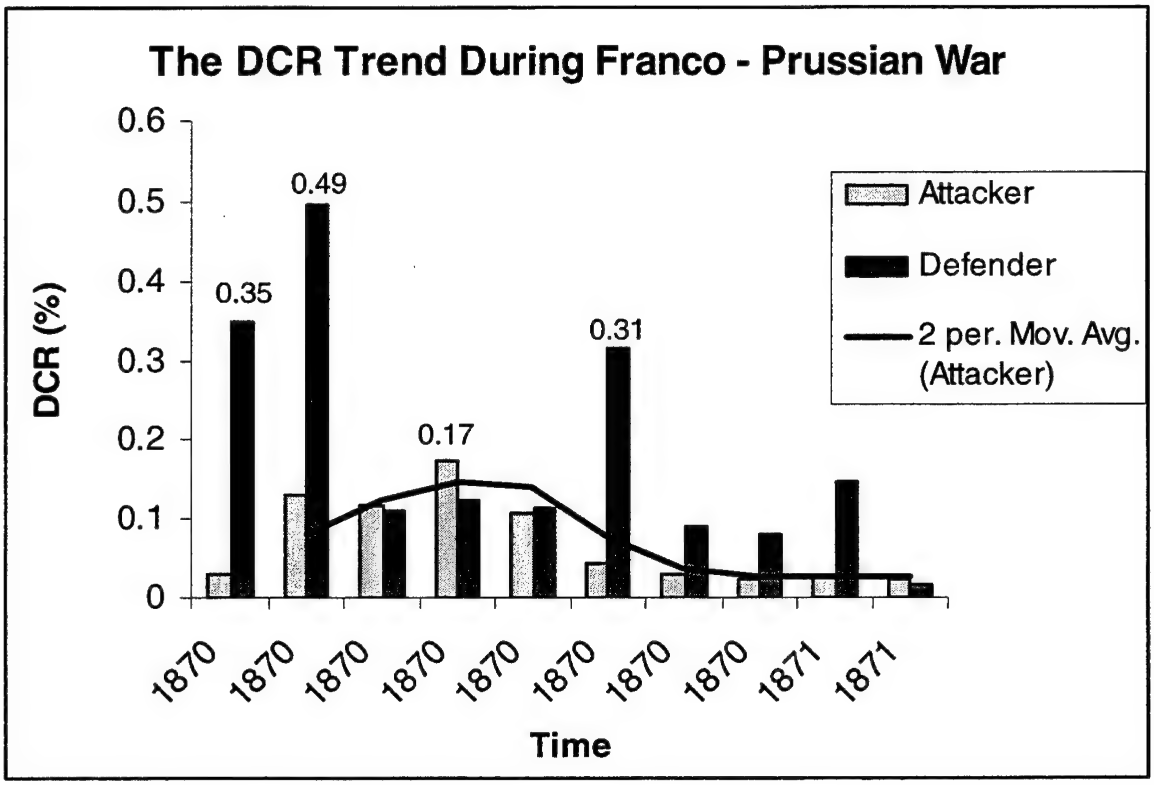

Figure 5.24. The Change in the DCR Values during the Campaign.

13. World War I (WWI)—(1914-1918)

WWI lasted for four years, from the marshlands of Eastern Europe to the deserts of Africa. More than 40 nations were engaged in the battles in two opposing groups: mainly Austria-Germany against Britain-France-Italy.

The campaign was dominated by thousands of trench wars [Ref 3.3]. The battles in the campaign were conducted in a way that neither side used the mobility of the forces, except for a few cases. The attackers and the defenders preferred facing each other in

trenches to prevent the advancing of the opponent, rather than exploiting changes in the battlefield.

On the other hand, new weapons were introduced to the battlefield in WWI. The Germans used chlorine gas [Ref 3.3] for the first time in history as a means of assault in 1915. The Germans surprised the defending French forces and advanced freely through French defensive lines for six miles.

Tanks [Ref 3.3] were initially designed as a solution to the problems of positional warfare, and later adapted as a means of exploitation. Tanks did not play a significant role in the German breakthrough in 1918, but they were part of the combined arms scheme which enabled British, French, and American forces to launch a series of successful offensives in 1915.

Despite these new weapons, the average DCR values kept decreasing. It is important to mention that the average DCR value of the attacker falls below 5%, 0.049, while the average DCR value of the defender is below 15%, 0.11. Also, in WWI the average unit sizes of the attackers and the defenders hit six-digit numbers: 199,126 and 140,222 [Table 5.13].

World War - I 1914						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
8.266129	199126.3	140222.1	43174.11	38716.22	0.049906	0.113819

Table 5.13. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The unit size comparison of the DCR values preserves the trend that the DCR values decrease as the unit size increases. The U.S. 2/28 Infantry Regiment of 1150, for example, totally destroyed the German 2/396 Infantry Regiment of 400 in St. Amand Farm in 1918 [Figure 5.25]. However, the French army of 1,000,000 and German force of 480,000 engaged in Aisne in 1918 inflicting about 5% casualty on each side.

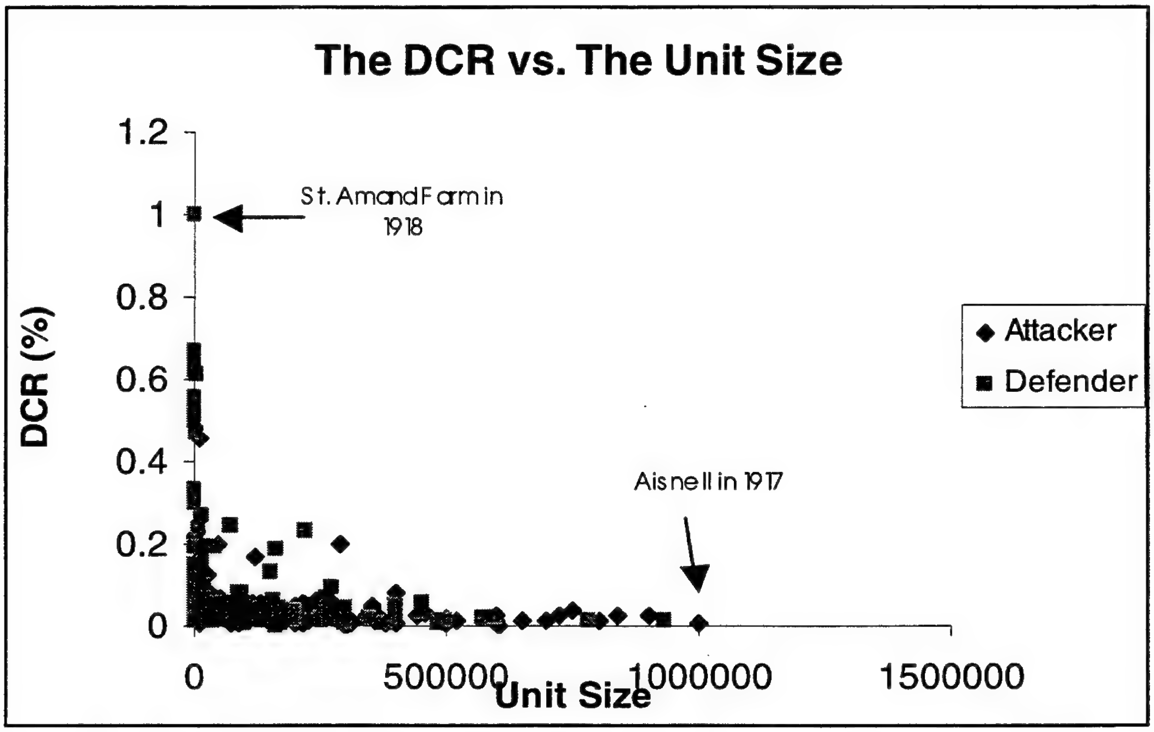

Figure 5.25. Change of the DCR Values with respect to the Unit Size.

The bloodiest battles of the campaign took place in the summer and autumn of 1918. This was the time when German forces tried to break the defense lines of the western front. Both the attackers and the defenders suffered heavy casualties attempting to dominate the battlefield. The French and British armies reinforced by fresh American forces contained German attacks and forced them to take defensive actions.

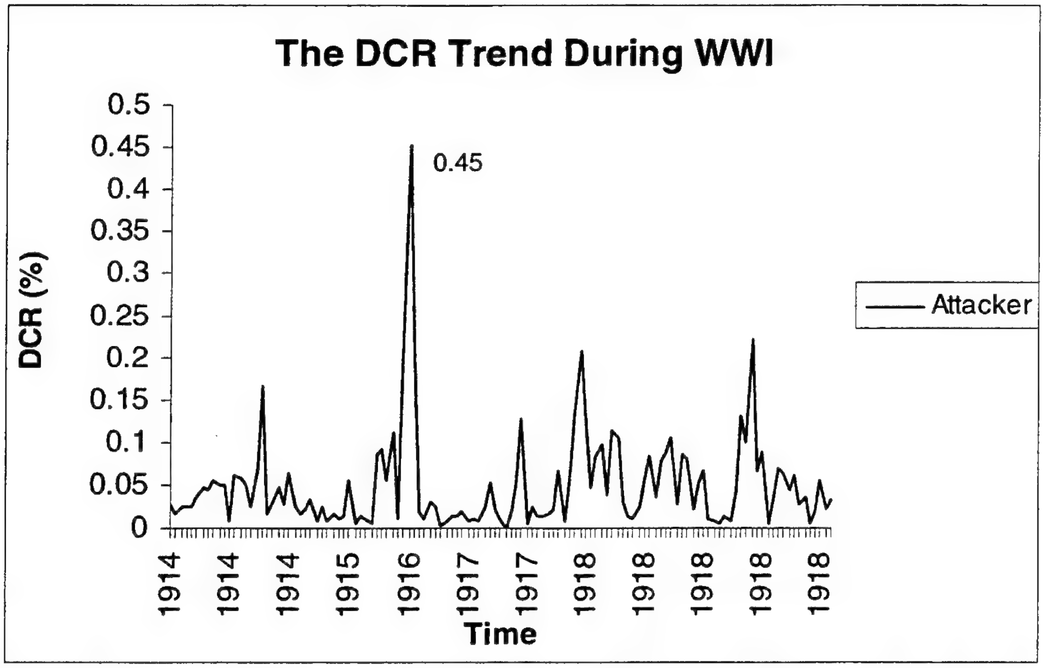

Figure 5.26a. (Attacker) The Change in the DCR Values during the Campaign.

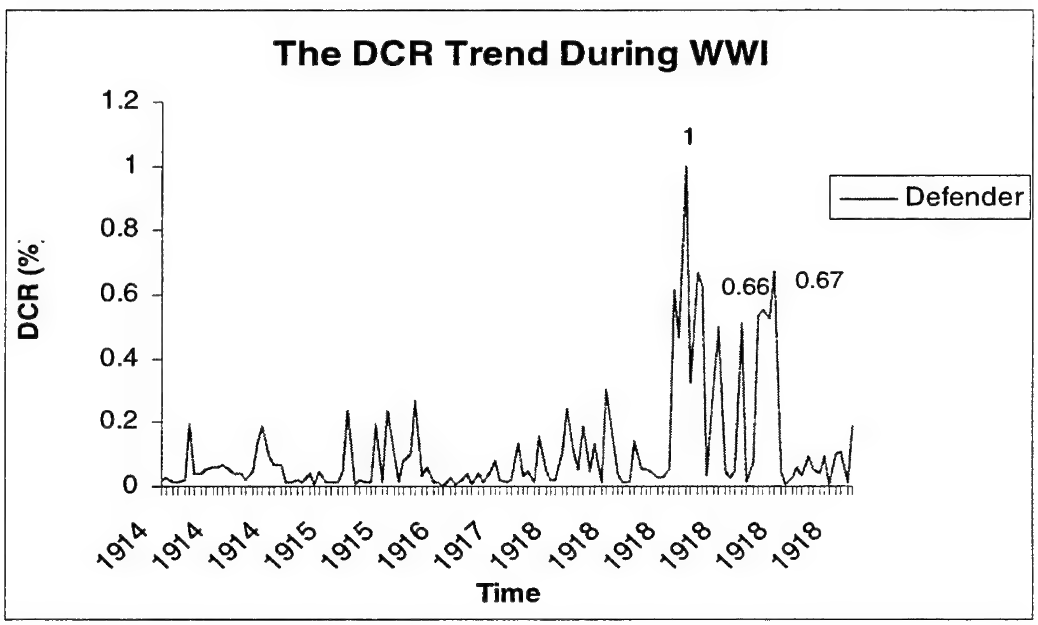

Figure 5.26b. (Defender) The Change in the DCR Values during the Campaign.

14. World War II (WWII)—(1939-1945)

In terms of the nations involved, the territory covered, and the destruction inflicted, WWII was the largest of all campaigns ever held. The striking success of the Germans at the beginning of the campaign was the product of a very transitory set of advantages. The Germans had produced equipment and fielded mechanized units in the mid-1930's, so this equipment was still usable and the units were well-trained and organized when the war began in 1939. In this period of the war, the Germans did not face any major opposition, and consequently, received minor casualties.

After the Normandy invasion [Ref 5.2], which was the largest combined land, sea, and air operation comprising 2,876,000 troops, bloody battles took place on mainland Europe between the Allied forces and Germany; in Eastern Europe between Russia and Germany; in Africa between the Allied forces and Germany; in the Pacific between Japan and the U.S. forces. Superior firepower generated by air force and ground forces were used to dominate the battlefield.

Despite the use of extensive firepower and the involvement of a huge number of personnel, the average DCR values of the attacker and the defender kept decreasing below 10%: to 0.01 and 0.06 [Table 5.14], respectively. Also, the difference between the DCR value of the attacker and that of the defender decreased [Table 5.14].

World War - II 1944						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
4.37234	83131.37	46566.24	6793.556	14714.94	0.017539	0.063255

Table 5.14. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The size of the unit continues to play an important role on the DCR values of the attacker and the defender. For example, 96% of the Japanese force was destroyed when the U.S. force of 5,237 attacked the Japanese force of 2,500 at Yaeju (Okinawa) in 1945. In addition, the Japanese force of 2,500 suffered a 0.58% loss by the U.S. attack of 16,002. However, Russian force of 2,200,000 troops suffered far less with 0.09% loss when they attacked German positions all along the 500-mile frontline at Vistula River in 1945.

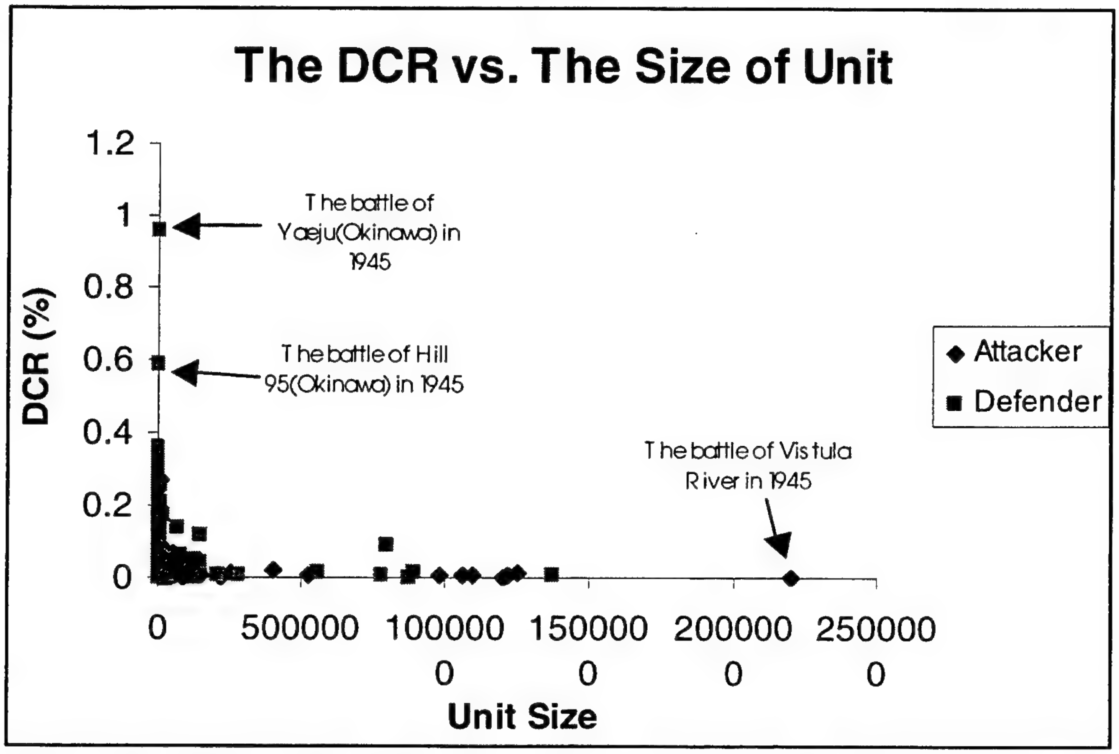

Figure 5.27. Change of the DCR Values with respect to the Unit Size.

The bloodiest battles of the war were conducted in Western and Eastern Europe and in the Pacific after mid-1944 [Figure 5.27]. The Allied forces engaged the German forces along all fronts in mainland Europe: Russians vs. Germans in Eastern Europe, Allied forces vs. Germans in Western Europe, and the U.S. forces vs. Japanese forces in the Pacific.

The DCR values of the attackers and the defenders fluctuated between 0.96 and 0.001 after mid-1944. However, the DCR values generally remained above 25%, which is close to the 30% level of destruction. Two periods during the war attract attention with relatively small DCR values: 1939-1943 and spring of 1944.

There were minor casualties in 1939-43 period since it was totally dominated by German victories facing little or no major oppositions. The spring of 1944 was spent as a preparation period on both sides of the conflict.

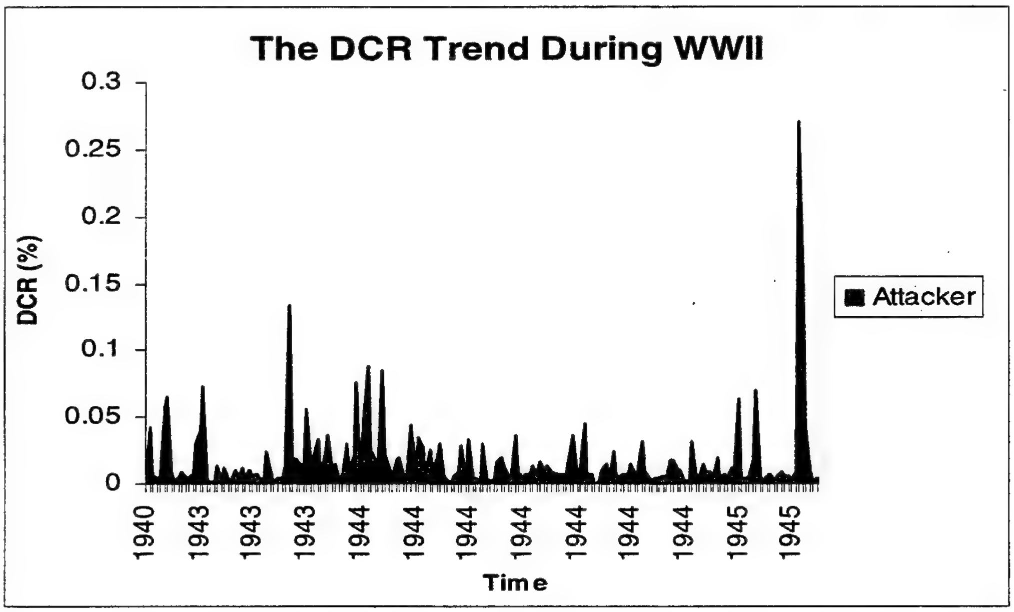

Figure 5.28a. (Attacker) The Change in the DCR Values during the Campaign.

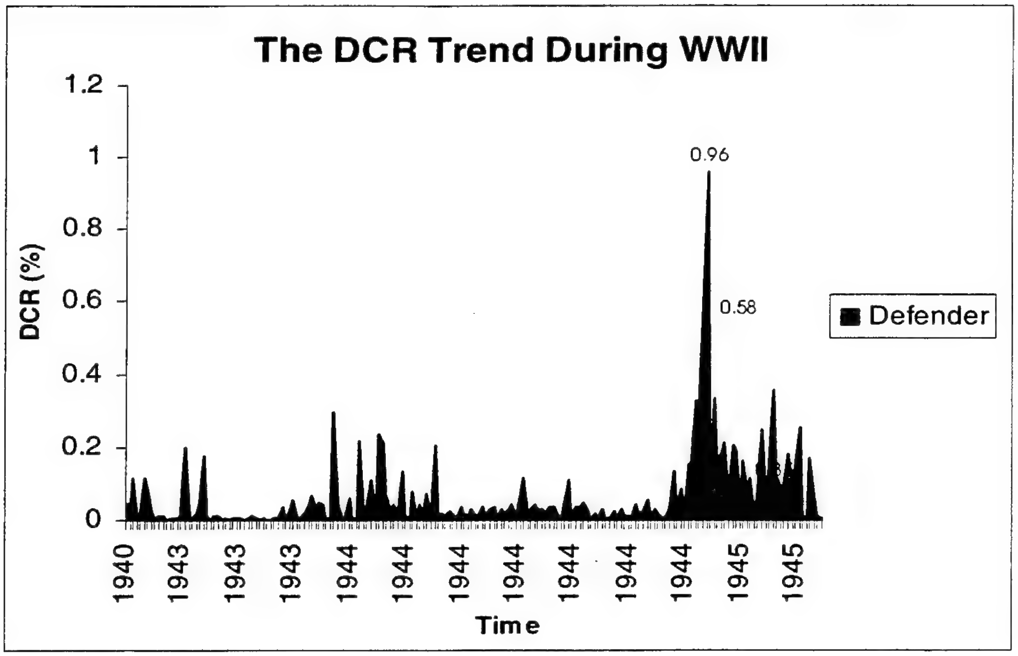

Figure 5.28b. (Defender) The Change in the DCR Values during the Campaign.

15. Arab–Israel War—1948

In many aspects, the Arab–Israel War in 1948 preserves almost the same features and trends including, the DCR as in WWII. Since the time frame of these two campaigns occurred so close to one another, the expertise acquired I WWII actually aided the Arab-Israeli War. One factor to consider [Ref 5.2], however, is that the Israeli forces were more likely to succeed against their enemies in the battlefield due to their superior discipline, high training level, and dominant firepower, especially from the air.

When compared to other historic battles, it is obvious that the average strengths of either the attacker or the defender in the campaign fall well below 5,000: 3,944 and 3,366 respectively. Despite a relatively vast difference between the Arab–Israel War and previous campaigns, in terms of the average strength values, the campaign preserves the general trend that the average DCR values decrease through history: 0.01 for the attacker and 0.05 for the defender [Table 5.15].

Arab - Israel War 1948						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
5.333333	3944.444	3366.667	298.3332	792.2221	0.014914	0.051235

Table 5.15. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

On the other hand, the unit size comparison of the DCR values has a reverse trend in the campaign: the larger the unit size, the larger the DCR value. Actually, small DCR values corresponding to large unit sizes are expected. Due to the unusual features of the campaign, such as the firepower superiority, even small Israeli forces inflicted large casualties on its Arab opponents. In the battle of Golan, for example, the Israeli force of 4,000 inflicted 11.7% DCR on the Syrian army of 6,000. The Israeli DCR value, however, was 5% [Figure 5.29].

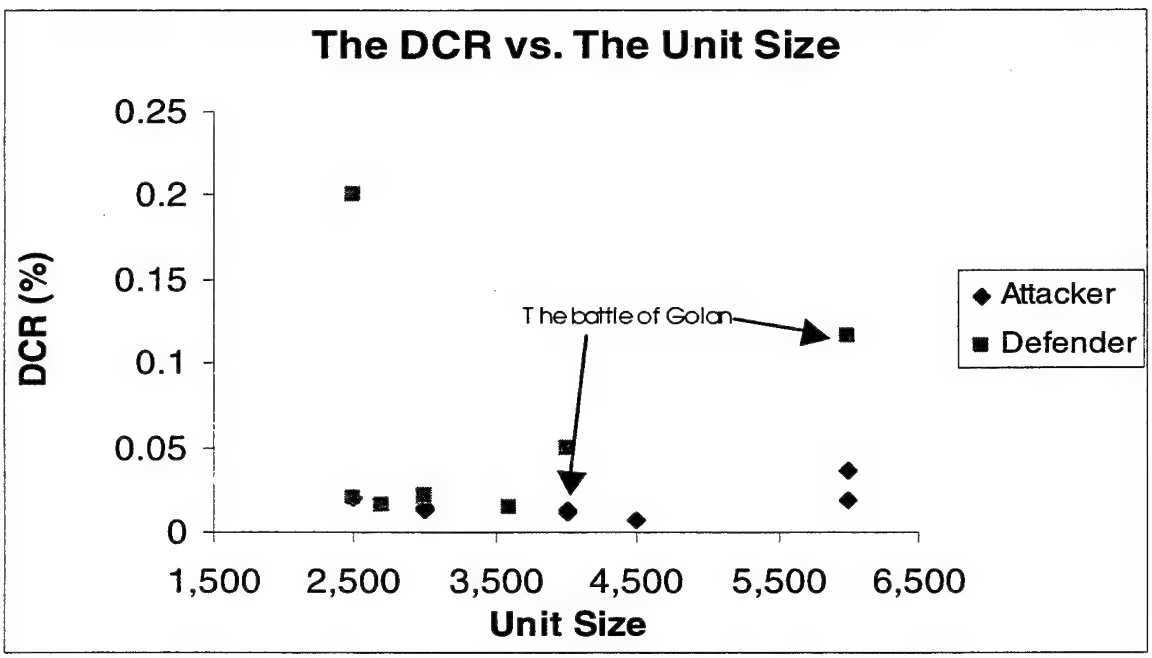

Figure 5.29. Change of the DCR Values with respect to the Unit Size.

The Arab–Israel War is assumed to be a biased case for combat modeling analyses due to the special conditions of the campaign. However, this campaign provides the most recent data concerning the combat modeling.

Another interesting feature of this campaign's DCR value is the DCR trend during the campaign. First of all, the Israeli forces were in an attacking posture for seven of nine battles. Israeli forces inflicted high CR's on their opponents as they received relatively low casualties [Figure 5.30]. Thus, the attackers' DCR, mostly Israeli forces, remained well below 0.05. As the campaign continued, however, the Arab army became accustomed to the Israeli method of conducting battles, so their DCR values, mostly as defenders, gradually decreased from 0.2 to almost 0.001.

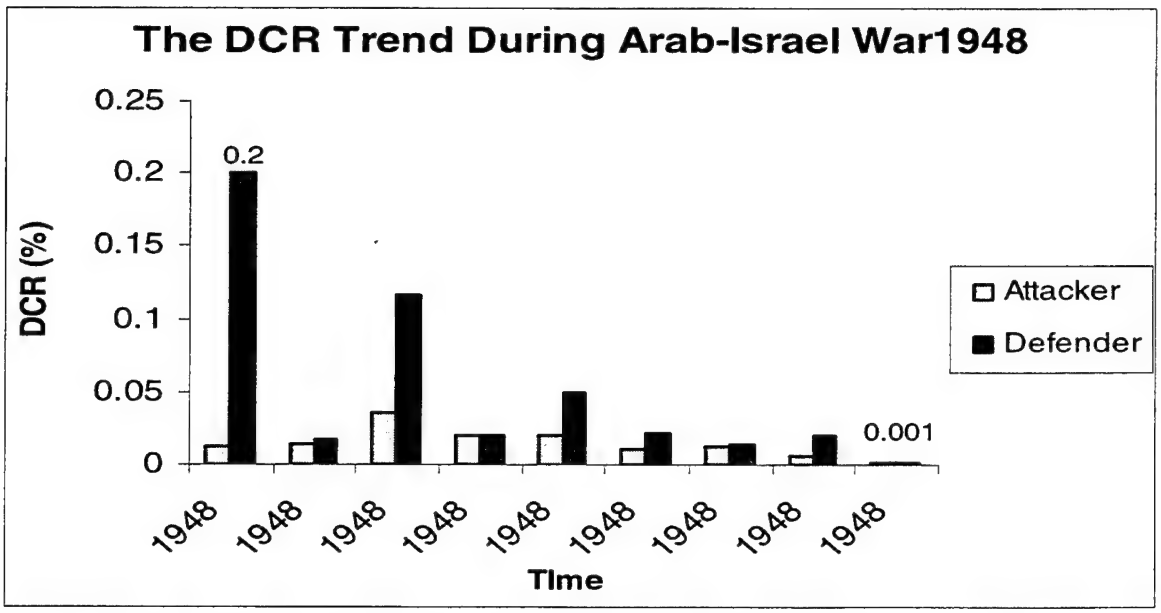

Figure 5.30. The Change in the DCR Values during the Campaign.

16. Korean War—(1950-1951)

The Korean War is the first group of battles fought under the U.N. umbrella [Ref 3.3]. The war was triggered by the Northern invasion of the South. The campaign can be divided into two groups of battles: the battles between 1950 and early 1951 and the

battles in 1951. In 1950 and early 1951, the Allied U.N. forces attacked in almost all of the battles, except for the battle of Pusan Perimeter, in which the North Korean army of 11,000 attacked the U.N. force of 15,000. Defender casualty rates with the highest value of 10% resemble those of the North Korean casualty rates.

Korean War 1950						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
3.545455	22727.27	20236.36	829.0909	3310	0.008275	0.039885

Table 5.16. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

In late 1951, however, the North Korean army attacked the U.N. forces in two of three battles. Also, the North Korean army received high DCR values, 0.037 and 0.19, as attacking forces [Figure 5.31]. The DCR values of the attacker and the defender remained below 10% level during the campaign.

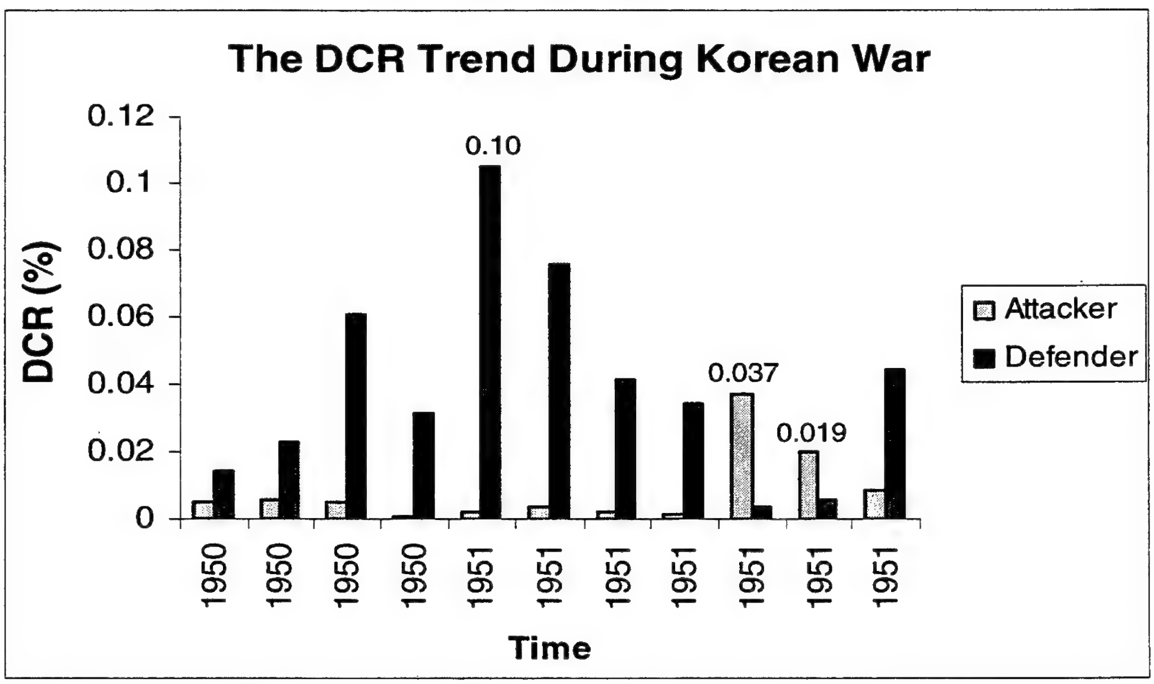

Figure 5. 31. The Change in the DCR Values during the Campaign.

One difference between the two periods of the campaign is the force sizes that were used in the battles [Ref 3.3]. The campaign until early 1951 was dominated by battles between relatively small units, which remained below 20,000 for both the attackers and the defenders during this period. In 1951, however, the unit sizes of the combatants in the battlefield gradually increased, hitting 37,000 in the battle of Iron Triangle in 1951.

The early period of the campaign preserves the trend that the DCR values decrease as the unit size increases, but roughly. The later period, which is associated with large units in the battlefield, however, does not reveal any trend concerning a relation between the unit size and the DCR value.

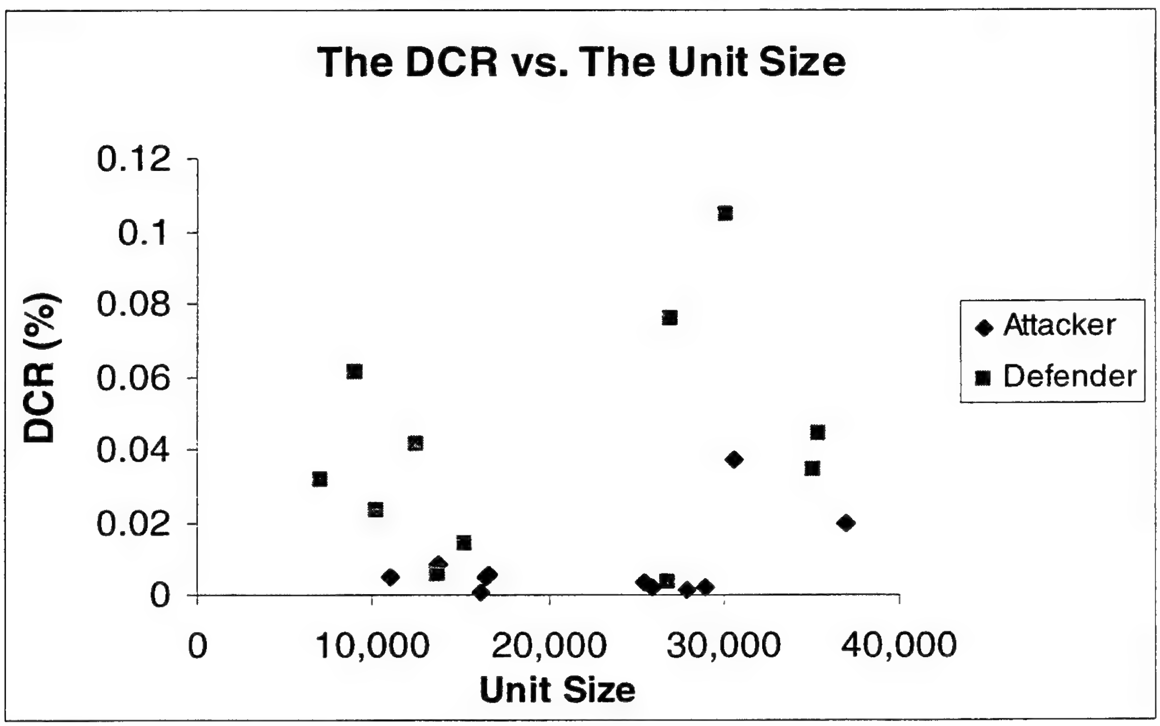

Figure 5.32. Change of the DCR Values with respect to the Unit Size.

17. Arab–Israel War—1973

When the Arab–Israel War in 1973 is analyzed, it is evident that the Israeli forces had consistent ground combat effectiveness superiority over the Arabs [Ref 5.2]. Also, the Israeli combat forces were supported by absolute air domination in the region during the campaign. Due to these conditions, the Israeli forces not only inflicted severe casualties on the Arabs but also could determine the results of the battles in which they were engaged.

However, these features of the campaign did not change the manner in which the DCR value continued decreasing through the modern era. The average DCR values of the attacker, generally Israeli forces, and the defender, mostly Arab forces, support the trend of the decreasing DCR value, even below 0.05: 0.01 and 0.02 respectively [Table 5.17].

Arab - Israel War 1973						
Average duration of a battle in the campaign t(days)	Average strength of the attacker A	Average strength of the defender D	Averge casualty of the attacker dA	Averge casualty of the defender dD	Average casualty rate of the attacker DCR_A	Average casualty rate of the defender DCR_D
2	20724.41	18864.59	478.2759	658.6207	0.013489	0.021981

Table 5.17. Average DCR Values, Strengths, Casualty Number, and Duration of a Battle.

The unit size comparison of the DCR values gives some clues that the average DCR values of the attacker and the defender decrease as the unit size increases. It is obvious that there are several DCR values corresponding to a single unit size value. The variation of DCR values corresponding to the same unit size in this campaign may contradict the trend that DCR values generally decrease as the unit size increases.

However, another point regarding the relationship between the DCR and the unit size should be examined, namely the upper limit of the change in the DCR value as the

unit size increases. The upper limit of the DCR values decreases as the unit sizes of the forces increase. As we analyze the DCR values of relatively large forces, we face relatively small values corresponding to the DCR values.

In the battle of Golan [Figure 5.33], for example, the attacking Israeli forces of 4,850 inflicted a DCR of 0.05 on the Syrian army. The DCR value was 0.008 for the defending Israeli forces when almost the same number of Syrian army attacked the Israeli positions. The upper limit of the DCR value corresponding to the force size of 4,850 becomes 0.05. On the other hand, the DCR value for the attacking Egyptian army of 81,000 in Sinai is 0.02, far less than the upper limit of the previous unit size.

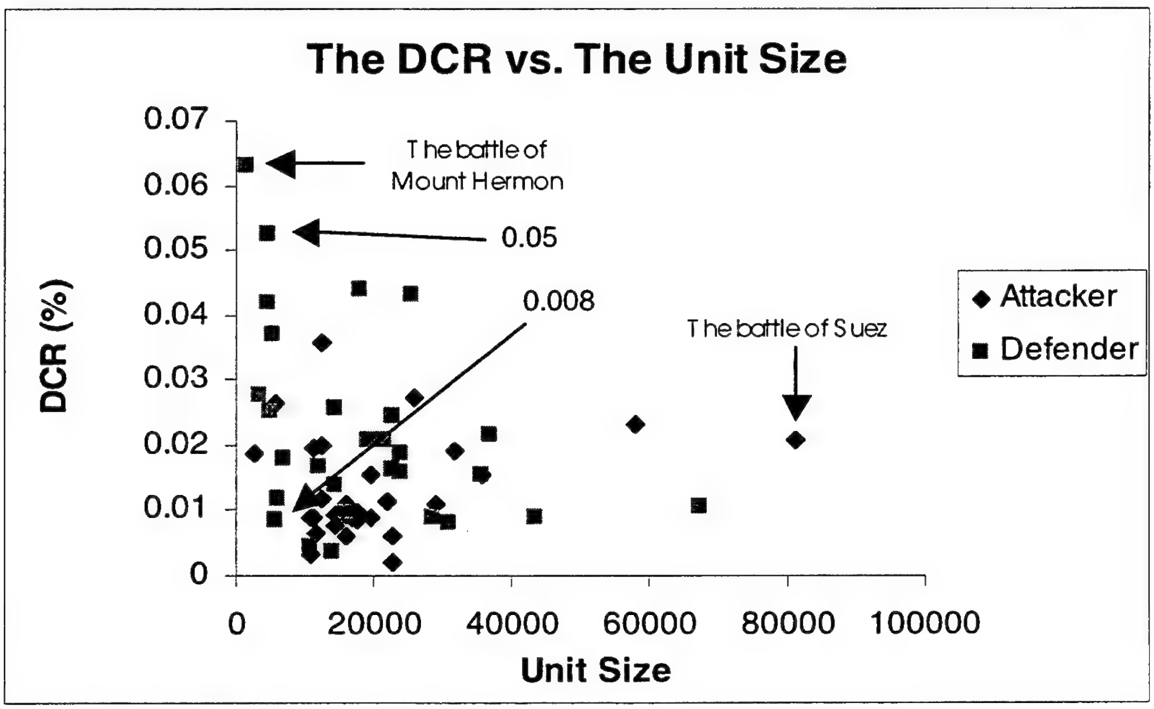

Figure 5.33. Change of the DCR Values with respect to the Unit Size.

The campaign is full of fluctuations in terms of the DCR values varying from 0.06 to 0.002. An important point is that all the DCR values of the attackers and the defenders are quite less than 7%. Yet the battles of modern warfare are expected to have small

casualty rates during the course of any battle. Since the Arab–Israel War of 1973 is the latest major campaign in the CDB90FT data set; it should be the one that has the smallest average DCR values of the attacker and the defender, which is the case here.

The highest DCR values of 0.06 and 0.05 occurred in the battle of Golan in which the Golani Brigade of Israel attacked the Syrian Paratroopers and the battle of Col Drori, in which the Israeli forces also attacked and inflicted casualties on the Syrian army. In both cases, the Israeli forces were numerically superior to the Syrian field army.

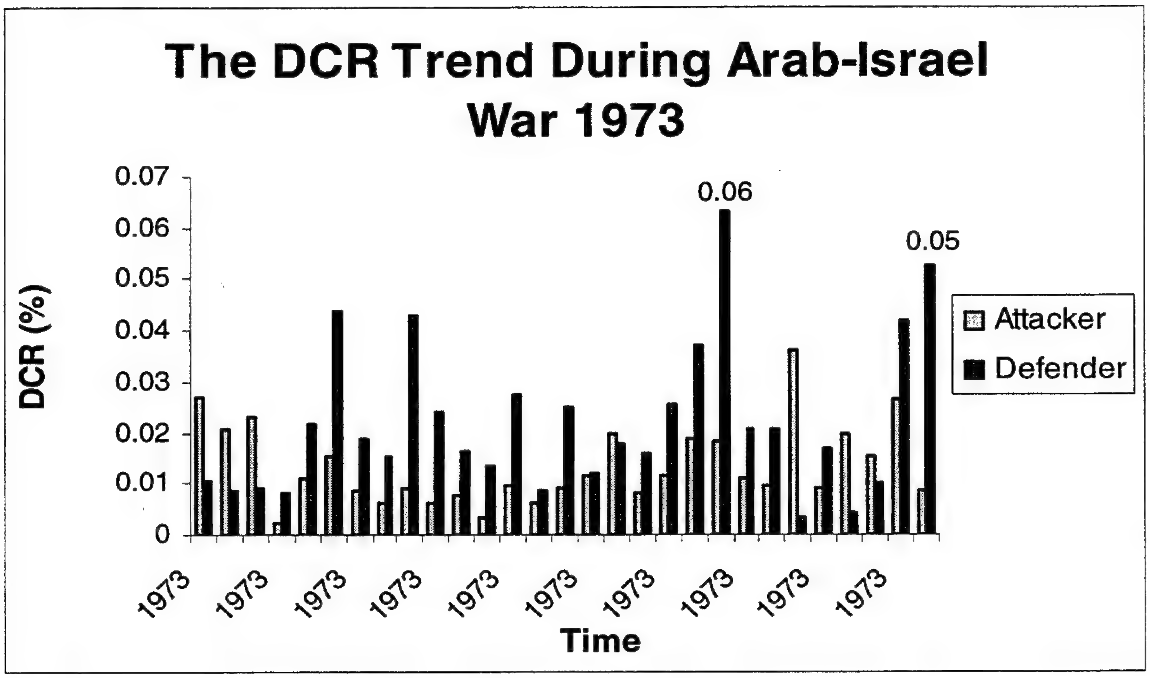

Figure 5.34. The Change in the DCR Values during the Campaign.

D. CONCLUSIONS

In the process of relating the trends of the DCR in the historical data to reasons behind those trends, figures are used to show how outcomes could be related to casualties. Actually, casualties varied so greatly in similar situations that it is necessary to look carefully to identify the correct historical patterns that would permit reasonable casualty predictions. Three very general patterns are evident in the historical casualty

data that has been analyzed. Casualty rates have declined generally over the past four centuries and almost leveled off at the rates experienced in WWII and the Arab–Israel Wars. The casualty rates of the attackers are almost always lower than those of the defenders. Also, the CR values decrease as the unit size in the battle increases.

During the 17th and 18th Centuries, the CR values declined for both the attackers and the defender, until the period of the French Revolutionary and the Napoleonic Wars. These examples provide specific evidence to support the general downward trend. However, the generally downward trend of the casualty rates reversed temporarily in two periods [Ref 2.2]. The first is the period of about 30 years including the Napoleonic Wars; the second is a period of similar duration encompassing the American Civil War and the Franco–Prussian War. It is useful to examine these two periods in more detail [Figure 5.35], since they suggest the possibility that there could be similar reversals in the generally downward trend of the CR values in the future.

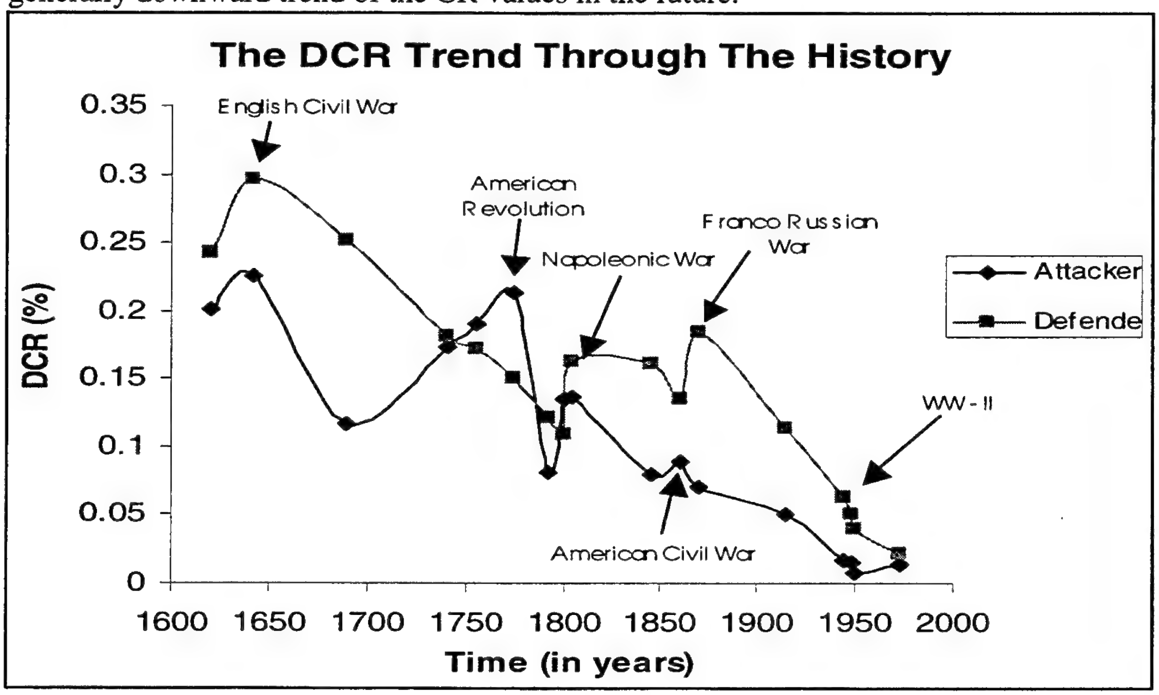

Figure 5.35. The Average DCR's of the Attackers and the Defenders in the Campaigns.

Grouped DCR Values		
Year	Attacker	Defender
1620-1695	0.181113	0.2633
1695-1805	0.1546732	0.149302
1805-1870	0.072605	0.148604
1871-1920	0.049906	0.113819
1920-1973	0.0135543	0.044089

Table 5.18 Grouped Average DCR Values.

The decline in CR values [Table 5.18] for both the attackers and the defenders from the Thirty Years' War through the French Revolutionary Wars to the U.S.–Mexico War is interrupted by the higher CR values of the Napoleonic Wars [Ref 2.2]. Evidently, there are two principle reasons for this. One reason is that Napoleon's enemies began to learn his method of warfare, which increased the efficiency of their battlefield performance, raising the CR values on both sides.

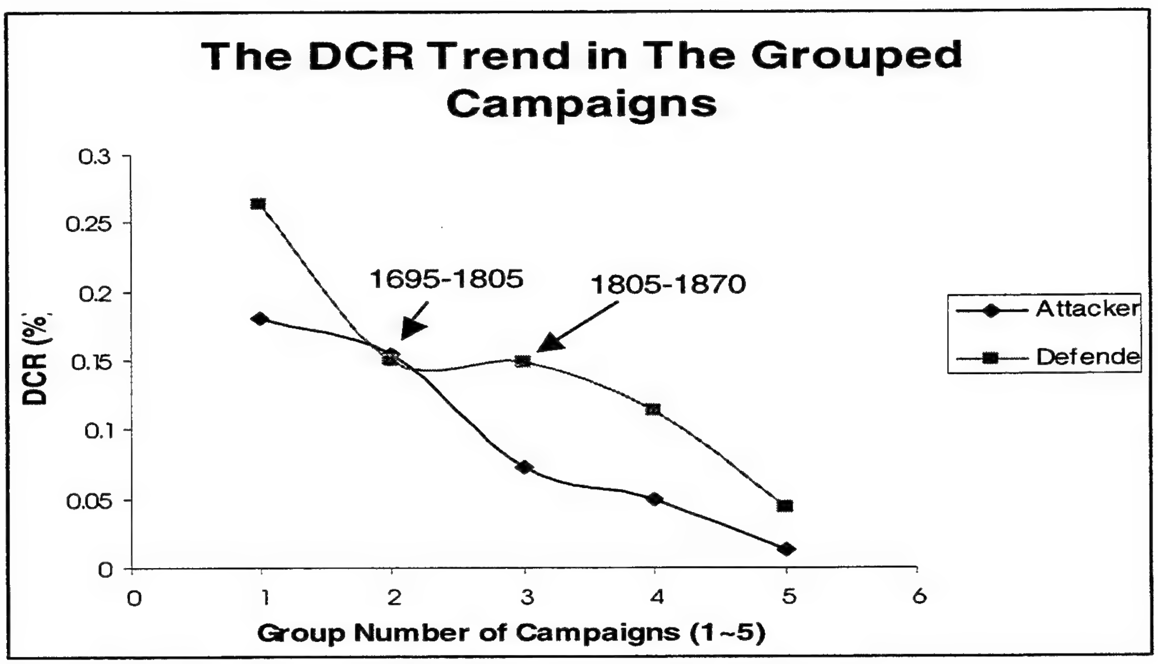

Figure 5.36. The Grouped Average DCR Values of the Campaigns.

The other reason is that these higher CR values caused a general decline in the quality of the forces that Napoleon led to battle [Figure 5.36]. This forced him and his opponents to rely on the mass attacks and to accept high CR's.

The reason for the increase in CR values in the period including the American Civil War was the introduction of the conoidal bullet [Ref 3.3], which replaced the long-range rifled muskets for the old, short-range, spherical ball firing muskets. This resulted in an improvement in the range, accuracy, and power of the infantry weapons, making the attacking forces more vulnerable to defensive fires.

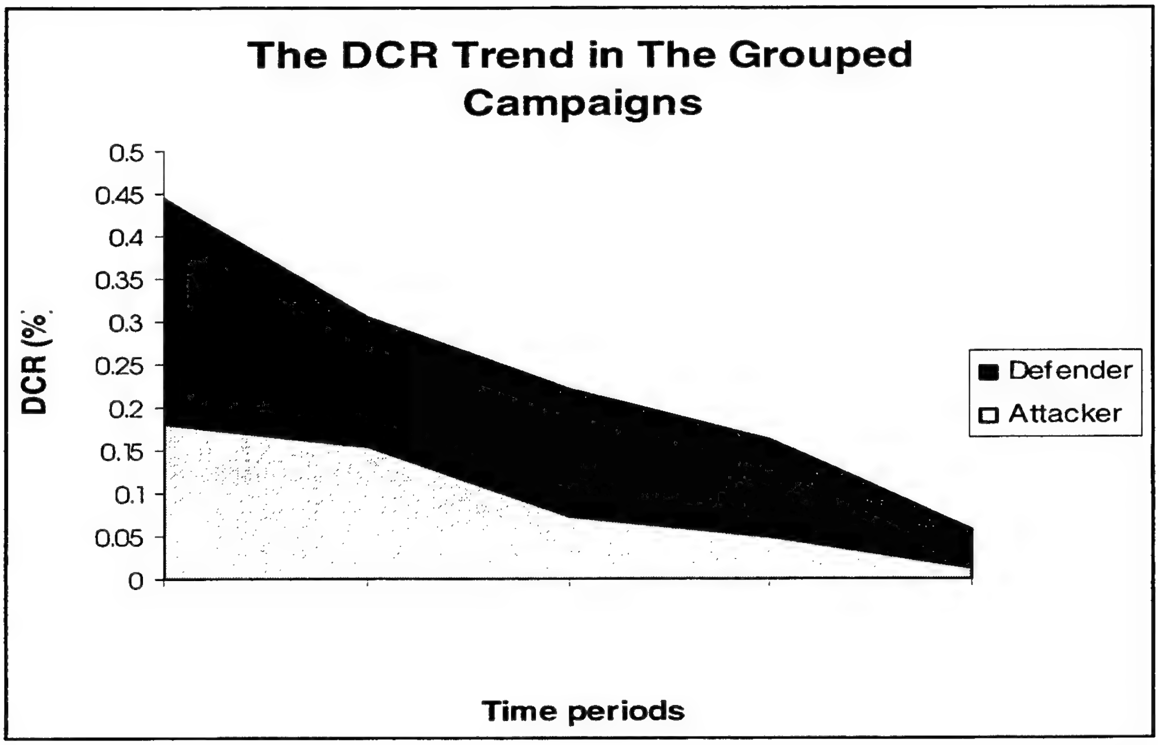

Figure 5.37. Cumulative Contribution of the Average DCR Values of Each Campaign.

Meanwhile, the DCR values of the small units are higher than those of the large forces under the same circumstances [Ref 2.2]. There are two principle reasons behind this. The first is that small combat forces have very few individuals who are not directly related to combat. Beginning with regiments and brigades, there are increasing numbers

and proportions of staff and support personnel and units who are not involved in actual combat activities.

The second reason [Ref 2.2] is the increasing amount of control over the units in the battlefield as the size of the unit increases. Thus, increasing delays in the performance of missions occur as the unit size becomes larger. To some extend, when large forces are engaged, there is an unintended cooperation of the opposing forces in the lower efficiency and lower CR values.

Even though CR values have declined steadily over the centuries [Figure 5.37], no guarantee exists that they will not rise again in the future. There are arguments that CR values in future wars will be higher than they have been in the past. However, the experience of historical combat does not support a significant reversal of the trend toward lower casualties.

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